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1970

Design requirements of precoat filters for water filtration

Harold Ray Bridges *Iowa State University*

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DESIGN REQUIREMENTS OF PRECOAT FILTERS

FOR WATER FILTRATION

by

Harold Ray Bridges

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY

Major Subject: Sanitary Engineering

Approved :

Signature was redacted for privacy.

In Charge of Major Work

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INTRODUCTION

History of Filtration

Filtration may be defined as the process of separating a mixture of solid particles and a fluid by passing the mixture through a porous medium which allows the fluid to pass through but retains the solid particles. The fluid may be liquid or gas and the solid particles may be of infinite variety. The desired product from this process may be either the removed solids or the filtered fluid. In the context of this thesis the desired product is water suitable for domestic use.

The word "filter" (fylter, filtre) is probably derived from the Latin filtrum, closely related to feltrum, meaning felt or compressed wool, and both are related to the Greek word, $i\pi\sigma\lambda$, signifying hair (24). The term "filtration" did not come into general use until the 16th century. Before that time, the words "sifting" and "straining" were used.

Actually, the "art of filtration" was probably practiced by ancient man long before the invention of the wheel since wherever man existed, at certain periods his drinking water must have become turbid, making some method of clarification necessary. Although there is no record of how man learned the simple principle, undoubtedly it was from repeated observation of some form of clarification as by the purification of water by trickling through sandy

soil or by the accidental passage of rainwater through an outstretched skin, garment, or tent cover. In any case man has apparently known the process since the dawn of history, far beyond the earliest records, wherein it was accepted as an established practice. References have been found to the manufacture of wine by the Chinese in about 2000 B.C. and it can be assumed that some kind of filtration was employed (24). At about this same period, persons in India were known to treat water by filtering it through charcoal, keeping it in copper containers, and exposing it to sunlight (24) .

The earliest (about 1250 B.C.) written record of filtration is that cut on the walls of the tomb of Rameses II, at Thebes, Egypt (24). There, illustrations depicted a kitchen scene with the drawing off of liquids of various kinds by means of threads. The earliest book which makes reference to filtration is Plato's "Symposium" (360 A.D.) in which capillary siphoning is also referred to. Additional references were made by Aristotle to the passage of water through earthen vessels and by Hippocrates, the "Father of Medicine", who advocated the boiling and filtering of polluted water before drinking (24).

After the fall of the Roman empire and throughout the Middle Ages, the art of filtration practiced by the ancient Egyptians and advanced by the Greeks and Romans was forgotten.

Some filtration was carried on by alchemists who generally employed capillary siphoning. Sand filters similar to the type used in modern practice weren't developed until the 18th century. What is generally thought to be the first filter patent ever issued was that granted to Joseph Amy by the French Government in 1789 (24). This called for downward filtration through sand or sponge in a vessel having a false bottom. The first British filter patent was granted to Peacock in 1791. Peacock described his invention as a new method of filtration by ascent through coarse gravel followed by graded sand which today is called upflow filtration. This method of filtration is even today receiving considerable research interest. Several other current research topics were mentioned in early patents. For example, in 1884 Isaiah Smith Hyatt took out a process patent on simultaneous coagulation and filtration. The basic principle of this patent involved doing away with the necessity of presedimentation and pre-coagulation of water prior to its filtration.

Modern public water filtration dates from 1829, when James Simpson built the first slow sand filters for the Chelsea Water Company of London. The use of slow sand filters was at first slowly and then rapidly adopted. Two incidents led to the rapid adoption of sand filters. First, John Snow gave epidemiological proof that the London cholera

epidemic of 1854 was traceable to contaminated and unfiltered water drawn from the Broad Street well. Second, in 1892 Dr. Robert Koch traced the cholera epidemic in Hamburg, Germany, to its unfiltered raw water supply. He did this by observing that the city of Altona on the opposite bank of the Elbe, which used the same water, but filtered it, had significantly less disease.

The first slow sand filters in the United States were built in 1872 at Poughkeepsie, New York by James Kirkwood who had previously traveled to Europe to study water filtration practices there. Shortly thereafter Patrick Clark, superintendent of the Rahway, New Jersey, water works installed a small rapid sand filter. Rapid sand filtration experiments at Louisville by George W. Fuller in 1895-97 showed that rapid sand filtration was successful if preceded by proper coagulation and sedimentation. Since Fuller's historic work, rapid sand filters have become firmly established in their use and major design features. Sand filter design into the early I960's was based principally on past experience and only in recent years have efforts been made to put the design of filters on a scientific basis. The ultimate goal of these efforts is to develop methods which can be used to optimize the filtration process.

In order to optimize successfully the design of a particular filtration process to give the maximum amount of

acceptable filtrate per unit cost, it is necessary to develop a mathematical theory which fully describes the process. This theory is needed to relate such filtration variables as head loss, filter run length, filtration rate, influent water temperature, and the characteristics of the suspended solids in the raw water. Today, there is considerable research interest in developing such a theory or theories for rapid sand filtration which is used extensively for the filtration of public water supplies. To date there is no theory which is generally applicable for rapid sand filtration, therefore optimum design of rapid sand filters has not been achieved.

During the past fifteen years considerable research has been conducted at Iowa State University on both rapid sand filtration and diatomite or precoat filtration. This research has led to the development of a theory of precoat filtration and an appropriate design method by which the design and operation of precoat filters may be optimized. The primary purpose of this thesis is to present this design method in a form which is readily usable to a design engineer and to define clearly the limitations and applicability of the theory of precoat filtration.

Precoat Filtration

General

Precoat filtration is a term applied to the process of filtration employing a thin (approximately 1/8-inch) layer of filter medium or filter aid. This process is a three-step operation. First, a thin protective layer of filter aid called the precoat (hence the term precoat filtration) is built up on the filter septum by recirculating a slurry of the filter aid (see Figure 1). After precoating, the filtering step is started. A small amount of filter aid called body feed is added to the incoming water. As the body feed is deposited at the filter surface a new filtering surface is formed. This prevents the formation of an impervious mat on the surface of the filter medium by the impurities removed from the water. After a predetermined head loss through the filter is reached, the filter is backwashed. The precoat is removed along with the body feed and impurities from the filtered water.

Filter equipment

Various types of precoat water filters are available. They usually fall into two general classifications depending on how the driving force is applied across the filter: pressure filters and vacuum filters.

Figure 1. Cross section of a precoat filter

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^{2} \left(\frac{1}{\sqrt{2}}\right)^{2} \left(\$

Pressure filters As the name implies, the prefilt is forced through the filter by either a pump in the influent line or by an available hydrostatic head. The pressure in the filter cake is designed to be greater than atmospheric pressure and the pressure differential across the cake is unlimited.

The most common types of pressure filters used in water filtration are cylindrical element filters and vertical leaf filters (Figure 2). These differ according to the shape of the filter septa (cylindrical or flat) and the manner by which the filter is backwashed.

The cylindrical element filters consist of vertical cylinders (about 1 to 3 in. diameter is typical) fastened to a tube sheet or header at the top (Figure 2(a)). They may be backwashed by reversing the flow of water or by "air bump" backwash. In this case, air is trapped on the filtered water side of the filter septa. At the end of the filtration cycle, the effluent valve is closed and the air is compressed to the maximum operating pressure of the filter pump. When the filter drain valve is opened, the air expands rapidly, forcing the water back through the septa with explosive force, thereby effectively loosening the cake from the septa. A variation of this is "multiple air bump" which was used by the U.S. Army Engineer Research and Development Laboratories (ERDL), Fort Belvoir, Virginia, in their

Figure *2,* Precoat filters

a) Pressure filter - cylindrical elements

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Figure 2 (Continued)

(b) Vacuum filter - vertical leaf elements

mobile treatment units during World War II. In this system, air is also trapped on the raw water side of the filter septa. Immediately after opening the filter drain valve, the air on the raw water side is suddenly released, forming a pocket into which the backwash water can surge before draining out.

Vertical leaf filters provide a flat filtering surface as opposed to cylindrical element filters. All vertical leaf filters are basically the same as far as precoating and filtering procedures are concerned, but they differ in the method of cake removal. These filters may therefore be further classified by the method of cake discharge as dry discharge filters and wet discharge filters. With dry discharge filters the filter leaves are either pulled out of the shell, or the shell is pulled away from the leaves. The cake can be removed as a wet mud by mechanical scraping or as a slurry by manual sluicing. Wet discharge filters are equipped with an internal cake sluicing device, thus eliminating the need for opening the filter for cake removal. A system of water jets is generally used in a manner which peels off the cake from the leaves, breaks it up, and flushes it out of the drain. Another type of vertical leaf filter is the reversible flow - wet discharge filter. These filters are designed so that by reversing the direction of flow, the spent cake can be removed from

one side of the filter element while the opposite side is being precoated. During this operation, the filter discharge is directed to waste.

Vacuum filters In the vacuum filter, the filter is located on the suction side of the pump so that the pressure on the influent side of the filter is at atmospheric pressure. This allows filtration to be performed in an open tank where the filter elements can be seen at all times. The principle disadvantage of vacuum filters is that the driving force (pressure differential) across the filter is limited to the vacuum that can be pulled by a normal pump (about 18 to 22 ft of water). Therefore, the vacuum filter is limited to use with raw waters of relatively good quality where little driving force is required for long filter runs. The pressure filter is indicated where a heavy suspended solids load must be removed and a larger driving force is required to provide a reasonable length of filter run.

As with the pressure filter, the septa of a vacuum filter can be either of the cylindrical or the vertical leaf type. The vacuum filter can also be cleaned by either manual or automatic sluicing.

A recent development in precoat filtration is the rotary vacuum filter. This filter is similar in construction to the vacuum filters used to dewater sewage sludges. For precoat

filtration, a thick precoat is laid on the filter and during the filtration cycle the filter cake and a small amount of the precoat layer are continuously removed by a rotary knife. This allows long filter runs to be made with a low driving force across the filter. In one installation, Hutto (39) reports the use of a 7-cm thick precoat and a knife advance rate of one cm per day so that a filter run would last for a full week.

Filter operation

Precoat filters can be operated under three conditions depending on how the pressure is applied across the filter and/or how the flow rate through the filter is regulated. These conditions are referred to as constant-rate filtration, constant-pressure filtration, and declining-rate filtration.

Constant-rate filtration Constant-rate filtration is most commonly used in current water works practice. In this method of operation, a constant pressure is usually supplied to the filter system and the flow rate through the filter is held constant by a manually operated or automatic flow control valve on the effluent line. Thus, as the filter resistance increases during a filter run, the pressure loss or driving force across the filter increases in order to maintain a constant rate of flow according to Equation 1:

Rate of flow =
$$
\frac{\text{driving force}}{\text{filter resistance}}
$$
 (1)

This is shown diagrammatically in Figure 3.

When the head loss through the precoat and filter cake reaches a value equal to the available pump head minus minor losses and the loss through the control valve when completely open, the constant rate run must be terminated. Since any further increase in filter resistance cannot be compensated for by the control valve, the ratio of driving force/filter resistance (Equation 1) will decrease and the flow rate must also decrease.

Constant-pressure filtration In constant-pressure filtration, the total available driving force is applied across the filter throughout the filter run and no provision is made to regulate the flow rate through the filter. Thus, the initial flow rate is equal to the total available driving force divided by the resistance of the precoat layer. As the filter run continues, the filter becomes clogged with solids and the filter resistance will increase. Therefore, since the available driving force remains constant, the flow rate will decrease. Actually, as the flow rate decreases, the minor head losses through the pipes, etc., will decrease and thus make a greater head loss through the filter available. These minor losses can generally be neglected

Figure 3. Head loss versus time for constant-rate filtration

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Figure 4. Head loss versus time for constant-pressure filtration

 $\sim 10^{-11}$

 $\Delta \sim 10^{11}$

 $\sim 10^{11}$

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for filter systems designed to operate at constant pressure. Also there may be an increase in the available pump head since most centrifugal pumps show an increase in head as the flow rate through the pump decreases (7). This type of operation is shown in Figure 4.

The constant-pressure method of filtration is seldom used in water filtration practice. Piping and all other appurtenances must be designed to carry the large volume of flow during the initial stages of a constant-pressure filter run. In addition, provision is required for relatively large volumes of water storage on both the upstream and downstream sides of the filter. These considerations have made constant-pressure filtration on a large scale uneconomical. This method of operation also makes difficult the addition of a constant proportion of body feed to the filter influent.

Declining-rate filtration Declining-rate filtration is a special case of constant-pressure filtration in that the total available driving force is applied across the entire filter system. It differs from constant-pressure filtration due to the fact that the filter influent and effluent piping is designed so that the associated head losses are not negligible compared to the loss through the filter itself. Therefore, at the beginning of a filter run, the piping will provide most of the head loss in the system

since the clean filter will have little head loss (Figure 5). Thus, the piping losses will control the flow rate early in the run. As the filter becomes clogged, the pressure loss through the filter will increase to a point where it overshadows the losses in the piping and the filter losses will gain control of the flow rate (Figure 5). The decrease in flow rate will be more gradual than in constant-pressure filtration because as the loss through the piping decreases, more head is made available across the filter. As with constant-pressure filtration, the filter run would be terminated when the flow rate becomes too low to satisfy requirements.

Declining-rate filtration is usually limited to small, non-municipal water filtration systems (small swimming pools) where no provision is made for automatic flow control needed for constant-rate operation and it is not desirable to handle the wide range of flow rates obtained with constantpressure filtration.

The preceding methods of filter operation produce the flow rate patterns shown in Figure 6. Constant-rate filtration is almost solely used for precoat filtration of water due to its inherent advantages of providing economy in the design of filter influent and effluent piping and the ease of adding body feed in constant proportion to the filter influent.

Figure 5. Head loss versus time for declining-rate filtration

Figure 6. Rate of flow curves for the three methods of filter operation

 $\sim 10^{-11}$

Filter aids

The first material used as a filter aid for precoat filtration of water was diatomaceous earth or diatomite. Prior to the introduction of other filter aid materials, the terms diatomite and D-E filtration were used rather than precoat filtration. Diatomite filtration is still a most common term although precoat filtration is more generally applicable.

Diatomite is composed of fossil-like skeletons of microscopic water plants called diatoms, members of the Bacillariophyceae class of algae. In the geological past of 15 or more million years ago, over 10,000 species of diatoms flourished in the waters covering certain of today's coastal areas. When these diatoms died and their skeletons sank to the ocean floor, large deposits of almost pure silica were formed. Later the land rose from the ocean floor and the deposits are now mined in open quarries. The largest and purest deposit of diatomaceous earth is located near Lompoc, California. Other deposits are mined along the western coast of the United States and Canada and throughout the world. The United States is the world's largest producer and user of diatomite. U.S. production during 1960-62 averaged more than 482,000 short tons per year, valued at about 24 million dollars (57).

Diatomaceous earth has many applications: as a filter

aid in the filtration of sugar syrups, beverages, and various chemicals as well as water; as a mineral filler in lacquers and paints, polishes, plastics, paper, insecticides, etc.; as high-temperature insulation; as an admixture for concrete; as an absorbent; and for countless other industrial applications (42). Processing the crude diatomaceous earth for use as a filter aid includes grinding, drying, and flux calcining. When flux calcining takes place, 3 to 10 percent by weight of either soda ash, sodium chloride, or caustic soda is added to the crude ore. Calcination affects the filtering properties of diatomite by changing the surface texture, agglomerating fines, and converting clay minerals to aluminum silicate slag (57). The slag particles are then largely eliminated in later processing steps. During the processing, the diatomite is separated into different particle size fractions by air classification. Particle sizes of individual diatoms vary from under 5 to over 100 microns. Grades of diatomite separated by air classification have mean particle sizes ranging from about 14 to 25 microns (55).

The performance of diatomite as a filter aid depends on the unique physical structure of the diatom particle (Figures 7 and 8). The almost infinite variety of shapes and sizes and the extremely porous framework of the skeletons provide numerous microscopic waterways and microscopic sieves which, when used as filter aids, serve to trap impurities.

Figure 7. An electron micrograph of an individual diatom from a typical diatomite, giving an indication of the straining potential of the media (4200X)

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 $\label{eq:2.1} \frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}^3}\frac{1$

Figure 8. A micrograph of a typical diatomite showing irregular diatom fragments (250X)

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Since the particles are rigid and strong, contact is limited to their outer points with the result that packing does not occur and the filter cake formed remains extremely porous. The porosity of a clean filter cake varies from 80 to about 90 percent for various grades of diatomite. Other typical properties of diatomite filter aids are found in Table 1 (42).

a
Registered trademark, diatomite filter aids, Johns-Manville Products Corporation, Manville, New Jersey.

Several other materials have been used as filter aids. Most successful of these is perlite, a material obtained by processing perlitic rock. Perlitic rock is composed essentially of aluminum silicate and contains 3 to 5 percent water (48). When crushed and heated, the rock expands and fractures to produce a light porous material similar to diatomite in both appearance and hydraulic characteristics. Perlite is used in many of the same ways as diatomite. As a filter aid, perlite is available in different grades which vary in both particle size and specific gravity. An average analysis of 10 perlites currently produced in 6 different states is given in Table 2 (48). A noticeable characteristic of perlite is that its bulk density in a filter cake is about one-half that of diatomite filter aids.

Like diatomite, perlite filter aids are produced in several grades of different particle size distributions. It has been found that there may also be differences in the characteristics of filter aid from various production lots of a particular grade and even from various bags of a particular lot (8). These differences arise from variations between deposits of diatomaceous earth or perlitic rock and the methods of processing the filter aids. Physical properties of several commercial filter aids are given in Table 3 (55). The values shown in Table 3 are averages of values obtained from tests with filter aids from several different production lots.

Table 2. Typical perlite properties

Physical properties ^a		Chemical properties
Specific gravity	$1.70 - 2.10$	0.20 Moisture loss at 105°C
Particle charge Bulk density in a filter cake, lb/cu ft	negative	Total moisture loss after ignition at 800°C 3.83
	$9.5 - 13.5$	Aluminum oxide $(A1,0,1)$, including any phos= phorous pentoxide or 13.08 manganese oxide
		0.72 Lime (CaO)
		0.89 Iron oxide (Fe ₂ O ₃)
		0.18 Magnesia (MgO)
		Potassium monoxide 4.44 (K_2O)
		73.20 Silicon dioxide (SiO ₂)
		3.31 Sodium monoxide (Na ₂ 0)
		0.04 Sulphur trioxide (SO_3)
		0.09 Titanium dioxide (TiO ₂)

^From laboratory tests conducted at Iowa State University.

Filter aids *S2, S3,* and 84 are perlite filter aids; all other filter aids listed in Table 3 are diatomite filter aids. The filter aid designations are explained in Appendix A.

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Defined on page 46.

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BACKGROUND AND LITERATURE REVIEW . Precoat Filtration of Water

The first extensive use of diatomite was for filtration of raw cane sugar liquor as early as 1876 (57). Today the primary industrial application of diatomite is as an industrial filtration medium for liquids ranging from municipal water supplies to alcoholic beverages. In contrast, substantial commercial production of perlite did not begin until 1946. In 1963 only 15 percent of the perlite produced in the United States was used as filter media (48). The major use was as an aggregate in building plaster.

The use of precoat filtration as a method of water treatment was not developed until World War II. During the Guadalcanal campaign, the U.S. Army found that military rapid sand filters were ineffective in removing cysts of Entamoeba hystolytica at the high filtration rates employed in the field (6-12 gpm/sq ft). These cysts are the causative agent of amoebic dysentery and are resistant to chlorination. Extensive research by the U.S. Army ERDL showed that diatomite filtration was effective in the removal of these cysts (14) .

The successful use of diatomite filtration during the war stimulated its application to civilian use, principally for the filtration of swimming pool water. However, due to

inadequate knowledge of the design and operation of these filters, many failed to provide an acceptable effluent at a cost comparable to sand filtration.

As a result of research concerning the basic principles of diatomite filtration and the development of better design and operating criteria, the use of diatomite filters steadily increased. In 1957, Phillips (58) effectively summarized what was then known about the design and operation of diatomite filters. His thesis contained an extensive literature review of diatomite filtration prior to 1957. Also in 1957, the American Water Works Association established a Task Group to determine more adequate design criteria for diatomite filters. In 1965, this group presented its report (67) which included a more current bibliography of the literature than Phillips' thesis.

In the thirteen years since 1957, much research in the precoat filtration of water has been conducted at Iowa State University under contract with the U.S. Public Health Service and, later, the Federal Water Pollution Control Administration. Eight Ph.D. theses and 15 master degree theses were completed during this time that are directly related to precoat filtration. Most of this research has been directed toward the development and evaluation of a theory of precoat filtration by which the design and operation of filters may be optimized (27, 43). Other research has been

concerned with the characteristics of different filter aids and grades (2, 26, 35, 49), the evaluation of the filtering properties of various suspended solids (3, 15, 37, 44, 61), and the use of polyelectrolytes and aluminum sulfate to improve the filtering characteristics of filter aids (16, 17, 20, 52, 56, 60). Extensive laboratory studies have been made to determine the applicability of precoat filters for the removal of iron (2, 35, 43, 50, 70), various flocculant solids (37, 44), clays (3, 20, 60, 61), and suspended solids from coagulated and settled surface waters (15), raw surface waters, softened waters (15, 27), and trickling filter effluent (36). A summary of almost all of the precoat filter runs made by these researchers is included in Appendix A.

Theory of Precoat Filtration

General

Since its conception by ancient man, the practice of filtration has developed as an art. Improvements in the "art of filtration" have been made by trial and error and from a series of successes and failures the present design of filters has evolved. The art of filtration is slowly becoming the science of filtration. Water pretreatment practices have been improved to the point where filtration is now considered as only a polishing step. Content with the knowledge that they can design filters that will work.

engineers are now trying to determine how filters work and how filters can be designed and operated at least cost.

In effort to determine how filters work, several authors have proposed various mechanisms by which suspended particles are removed within a filter. Burns (17) has presented an excellent discussion of the possible removal mechanisms involved in water filtration. Included are straining, gravitational forces, inertial forces, Brownian movement and particle diffusion. Van der Waals forces, electrical forces between surfaces, and chemical forces. In rapid sand filtration a considerable amount of the suspended solids are removed within the sand bed (i.e., depth removal) although in certain instances removal at the surface of the sand bed (i.e., surface removal) may predominate. Within a filter bed, all of the aforementioned removal mechanisms may occur, including interstitial straining near the point of contact(s) between filter grains. This has made the development of a single unified theory of rapid sand filtration a very complicated, if not impossible, task.

Precoat filtration is a form of cake filtration which is a fundamentally different process than fixed-bed granular filtration. In cake filtration the suspended solids are removed by straining at the surface of the filter to form a mat or cake of solids. Subsequent suspended solids are removed by straining at the surface of the previously formed

filter cake. In precoat filtration the filter consists of the supporting septum and the precoat layer. A grade of filter aid is chosen which is fine enough to strain the suspended solids to be removed mechanically. This is a special form of cake filtration in that filter aid is added to the influent as body feed. In effect, the addition of body feed may be considered as a method of pretreatment to reduce the flow resistance of the resulting filter cake.

The action of filter aid

Since filter aid particles are rigid and strong, they form a filter cake which is incompressible within the range of pressures encountered in water filtration (47). If enough filter aid is added as body feed so that pointto-point contact between particles of filter aid is maintained in the filter cake, the cake will be essentially incompressible. The specific resistance of an incompressible filter cake remains constant as the pressure on the cake changes. Therefore, for constant-rate filtration, the head loss through any previously formed portion of the filter cake remains constant throughout the filter run and, the only increase in head loss through the filter is due to the solids being removed at the surface of the filter cake. Now, the amount of solids (impurities plus body feed) removed per unit time remains constant during constant-rate filtration. Thus, the increase in head loss through the

filter per unit time will remain constant. This results in a linear head loss versus time curve as shown by Curve A in Figure 9.

Adding a greater proportion of body feed to the filter influent results in the formation of a filter cake which is more porous and therefore contains a greater volume of void spaces available for flow but also results in a thicker cake. Thus, the specific resistance of the filter cake may be less and the increase in head loss per unit time may be lower (Curve B, Figure 9).

If the proportion of body feed is lowered, eventually the point is reached where the filter aid particles do not make point-to-point contact when they are initially deposited on the filter cake. Essentially all of the impurities encountered in water filtration are compressible. Therefore, as the pressure on a layer of filter cake increases, the layer compresses until contact is made between particles of the filter aid. Compression reduces the volume of voids in the cake, increases the apparent specific resistance of the cake, and increases the head loss through the cake. The compressive pressure on any layer within a filter cake is equal to the pressure loss through the cake lying above that layer. Shown in Figure 10 is the distribution of compressive force within a filter cake at different times, t_1 and t_2 , during a filter run. As the filter cake

Figure 9. Head loss development curves for filter cakes with various amounts of body feed, flat septa

Figure 10. Distribution of compressive force within a filter cake at times t_1 and t_2

thickens, the compressive force on the layer of filter cake next to the precoat increases to the value where point-topoint contact is made between filter aid particles. After this time, the thickness of filter cake which is compressible remains constant and the thickness of the incompressible layer next to the precoat increases constantly throughout the filter run (Figure 10).

Compression of the filter cake results in an exponential rate of head loss increase (Curve C, Figure 9). Any further decrease in the amount of body feed increases the compressive pressure required to make point-to-point contact between filter aid particles. This increases the thickness of the compressible layer of filter cake and the exponential rate of increase in head loss (Curve D, Figure 9).

Finally, if no body feed is added, a head loss curve such as shown by Curve F in Figure 9 may result. Addition of a very small amount of body feed only adds to the thickness of the filter cake and increases the rate at which head loss increases (Curve E, Figure 9). One might expect that the maximum rate of head loss increase would occur when just enough body feed is added so that point-to-point contact within the filter cake is not made until the very end of the filter run.

Several authors have developed theories applicable to compressive cakes (19, 64, 68). However, the application of

these theories to precoat filtration of water would require the determination of compressibility factors of filter cakes with various amounts of body feed added, resulting in an extreme amount of difficult experimental work. Baumann and LaFrenz (10) have found that the optimum amount of body feed for least cost filtration produces a linear head loss curve and, therefore, an incompressible filter cake. For these reasons, the theory of precoat filtration has been developed for incompressible cakes only.

Precoat filtration equations

Darcy stated the basic concept for laminar flow through an incompressible porous bed in 1856 (23). Darcy's law states that the velocity of flow through a porous bed is directly proportional to the pressure gradient across the bed or:

$$
\frac{\mathrm{d}V}{\mathrm{Ad}t} = K \frac{\mathrm{d}P}{\mathrm{d}L} \tag{2}
$$

 $\Delta\sim 10^{-10}$ km $^{-2}$

where:

- $V = volume of filterate passing through the bed in$ time t $[L^3]$
- A = gross cross-sectional area of the porous media perpendicular to the direction of $flow$ [L²]

K = coefficient of permeability $[LT^{-1}]$

 dP/dL = pressure gradient [FL⁻³]

- $P = pressure$ loss across the porous media in the direction of flow **[FL~2]**
- $L =$ thickness of the porous media in the direction of flow [L]

The letters within brackets indicate the basic dimensions, force (F) , length (L) , and time (T) , of the above terms.

In later years considerable data have shown the rate of flow through porous beds to be inversely proportional to the viscosity of the fluid so that Darcy's equation is usually modified as follows:

$$
\frac{dV}{ddt} = \frac{K_1}{\mu} \frac{dP}{dL}
$$
 (3)

where

 K_1 = modified permeability coefficient independent of viscosity $[L^2]$

 μ = dynamic or absolute viscosity [FTL⁻²]

If the specific resistance to permeability on a volume basis, a, is defined as the reciprocal of the modified permeability coefficient, it is seen that the above modification of Darcy's law is the equivalent of Poiseuille's law for laminar flow through capillaries as presented in 1846 (59). Darcy's and Poiseuille's laws are both expressions of the basic flow relation given by Equation 1. The coefficient of viscosity is included as a correction factor to account for the effects of temperature.

LaFrenz (43) recognized that Darcy's law was applicable to flow through a precoat filter. He developed an equation for head loss development during a constant-rate filter run by applying the unmodified Darcy equation (Equation 2) to both the precoat layer and the filter cake. His equation was improved and published in the following form (5):

$$
H = Hp + Hc
$$

= K₃qw + K₄C_Fq²t (8.33 x 10⁻⁶) (4)

where :

 $H =$ total head loss through the filter (ft of water) H_p = head loss through the precoat layer (ft of water) H^{\prime}_{α} = head loss through the filter cake (ft of water) K_3 , K_A = constants including the coefficient of permeability of the filter cake and the bulk density of the filter aid in the precoat_layer and filter cake respectively (min ft^5/Ib gal) q = filtration rate (gpm/sq ft) $w = weight of precoat layer (lb/sq ft)$ C_p = concentration of body feed (mg/l or ppm) t = elapsed time of filtration (min)

The head loss through the precoat layer is equal to the initial head loss at the beginning of a filter run and was assumed to remain constant throughout the run (i.e., no suspended solids are removed within the precoat layer). The

term for the head loss through the filter cake was developed under the assumption that the thickness of the filter cake is determined by the amount of body feed and not increased by the suspended solids removed from the filter influent. This assumption is probably valid under the conditions for which Darcy's law may be applied (i.e., incompressible filter cake).

Because LaFrenz used the Darcy equation in its unmodified form, the coefficient of viscosity is not included in his equation. Therefore, the resistance coefficients K_3 and K_A are not true constants but vary with the temperature of the water. Baumann and Oulman (12) modified LaFrenz's equation to correct for changes in viscosity.

Dillingham (27) reviewed the theory of precoat filtration in 1965 and found that two factors had not been considered. First, at the end of the precoating operation the filter housing is full of clean water. Therefore, at the beginning of the filtering cycle the filter influent is diluted before it passes through the filter. This results in a transition period which lasts until the quality of the water in the housing is the same as that of the influent. During this period of "initial dilution", the rate of head loss development is lower than it is during the remainder of the filter run.

Second, when cylindrical septa are used, the outer

surface area of the filter cake increases as the thickness of the filter cake increases. This causes the flow rate per unit area of filtering surface to decrease throughout a filter run which has a marked effect on the head loss development during the run, especially when using small diameter septa.

Dillingham applied the modified Darcy equation (Equation 3) to develop a theory of precoat filtration which accounts for both the "initial dilution" and "increasing area" effects. The resulting equations are summarized in Table 4 (29). These equations may be used with any consistent set of units for the basic dimensions of force, length, and time. Units commonly used are pounds, feet, and hours.

	Equation	Equation number
Any septum	$H_p = qv \xi w / g$	(5)
Cylindrical septum	$H_c = \frac{R_s \sigma}{\phi} \ln(1 + \frac{R_s \phi X}{R_s})$	(6)
	$L = \sqrt{R_0^2 + R_s \phi X} - R_s$	(7)
Flat septum ^d	$H_c = \sigma X$	(8)
	$L = L_p + \frac{\phi X}{2}$	(9)

Table 4. Summary of precoat filtration equations

a_{Septum that does not exhibit increasing area effect.}

Table 4 (Continued)

 $\ddot{}$

 $^{\rm b}$ Dimensionless.

Table 4 (Continued)

 $\mathcal{L}^{\text{max}}_{\text{max}}$ and $\mathcal{L}^{\text{max}}_{\text{max}}$

Table 4 (Continued)

Symbol	Meaning	Dimension
δ	Dilution rate, theoretically Q/V_c	$[T^{-1}]$
ν	Kinematic viscosity of influent	$(L^{2}T^{-1})$
ξ	Filter aid resistance index or ξ index	$[F^{-1}L]$
σ	Arbitrary group of terms	$[LT^{-1}]$
φ	Arbitrary group of terms	$[LT^{-1}]$

A complete derivation of these equations is included in Appendix B. Their derivation is quite straightforward and includes the following assumptions:

1. Enough body feed is added to form an essentially incompressible filter cake.

2. Darcy's law applies (i.e., the flow is laminar).

3. Constant-rate filtration.

4. The outer surface area of the precoat layer is approximately equal to the outer surface area of the septa (i.e., thin precoat layer).

5. There are no concentration gradients in the filter housing (i.e., completely mixed system).

6. All solids, body feed and suspended solids, are removed at the surface of the filter cake (i.e., no solids pass through the filter and none are removed in the precoat layer).

7. The suspended solids retained in the filter cake do not increase the cake thickness appreciably.

8. The bulk densities of the precoat layer and filter cake $(\gamma_n$ and $\gamma_n)$ remain constant throughout a filter run. p^{and} q^{n}

9. The concentrations of suspended solids and body feed (C_c and C_F) remain constant throughout a filter run.

A filter cake is incompressible if enough body feed is added to provide point-to-point contact between filter aid particles in the cake. Point-to-point contact of filter aid particles would require that the suspended solids in the cake be deposited in the voids where they would not cause separation of the filter aid particles. Therefore, if the body feed rate is sufficient such that Assumption 1 is valid then Assumption 7 will also be met.

Since the channels in a precoat filter cake are very small, flow through them is considered laminar in accordance with Assumption 2. For flow in pipes and other large sections, the Reynolds number, which expresses the dimensionless ratios of inertial to viscous (or resistive) forces, serves as a criterion to distinguish between laminar and turbulent flow. Hence, by analogy, the Reynolds number has been employed to establish the limit of flows described by Darcy's law.

Reynolds number is expressed as:

$$
N_R = \frac{\rho v D}{\mu} \tag{10}
$$

where:

 $p =$ fluid density (γ_w / g) [FT²L⁻⁴] v = velocity of flow $[\mathbb{LT}^{-1}]$ D = diameter of pipe [L] μ = viscosity of fluid [FTL⁻²]

To adapt this criterion to flow in a filter cake, the apparent velocity or filtration rate, q, is used for v and an average grain diameter, d, is substituted for D. Rose (63) found that laminar flow in porous media exists at $N_R < 1$ and that turbulent flow exists when N_R >10. Thus, the maximum filtration rate at which the flow in a precoat filter cake is

laminar can be calculated as:
\n
$$
q = \frac{N_R^{\mu}}{\rho d}
$$
\n(11)

If
$$
N_R = 1
$$

\n $\mu = 2.359 \times 10^{-5}$ lb sec/ft² at 60° F
\n $\rho = 1.938$ slug/ft³

and

$$
d = 30 \text{ microns} = 98.4 \times 10^{-6} \text{ ft}
$$

then

$$
q = 0.124
$$
 ft/sec = 55.5 gpm/sq ft.

Obviously the flow is laminar at the filtration rates used in

precoat filtration (1-3 gpm/sq ft).

Assumptions *3, 4, 8,* and 9 impose conditions during a filter run which are generally met in practice. In certain cases such as filtration of a river water, however, the concentration of suspended solids may not remain constant during a filter run.

In order to describe the effect of initial dilution, it was necessary to assume completely mixed conditions within the filter housing (Assumption 5). It is very doubtful, however, if a filter has ever been built, or could be built, which achieved complete mixing. Just how well an actual filter housing approximates complete mixing has not been investigated.

It has been said that, "All filters pass some suspended solids all of the time." This is certainly the case in normal water filtration. Assumption 6, which states that no solids pass through the filter, was necessary in order to describe the effects of initial dilution. In this respect, the assumption is probably valid. However, it is assumed throughout the derivation that the specific resistance per unit volume of the precoat layer and filter cake both remain constant during a filter run. This implies that the precoat layer and filter cake are incompressible and that no solids are dislodged or deposited within the precoat layer or previously formed filter cake (i.e., no depth removal).

Since some solids are known to pass through a filter, it is likely that there are solids removed within the filter cake and precoat layer by interstitial straining or some other depth removal mechanisms. This is another possible source of error in the theory of precoat filtration that deserves investigation.

Prediction of filter cake resistance

In the precoat filtration equations, the filter cake resistance is indicated by the β index. The precoat layer resistance is indicated by the filter aid resistance index or ξ index.

Baumann et al. (5) reasoned that filter cakes containing equal ratios of suspended solids to body feed (C_S/C_F) should have equal specific resistance per unit volume (or weight) of cake. Therefore, specific resistance should vary only with changes in the type of suspended solids, grade of filter aid, or C_S/C_F ratio. Values of K_A were determined for several filter runs using University tap water to which ferrous sulfate had been added. It was found that a log log plot of $K_{\tilde{d}}$ versus C_S/C_F formed a straight line corresponding to the equation:

$$
K_{4} = a \left(C_{S} / C_{F} \right)^{D} \tag{12}
$$

where:

a,b = empirical constants The ability of Equation 12 to predict the value of K_d

was verified for several waters used in subsequent investigations by Hawley (37) , Hall (35), and Regunathan (61). Regunathan, however, found that this relationship did not fit results from the filtration of University tap water containing sodium montmorillonite clay mineral (Wyoming bentonite). Instead, the specific resistance was lower for higher values of C_S and C_F , even though C_S/C_F remained constant. This "solids concentration effect" was thought to be caused by the swelling properties of sodium montmorillonite.

From these studies it was evident that for a certain suspended solid, the specific resistance of the filter cake depended on the ratio C_S/C_F , the concentration C_F or C_S , and the grade of filter aid. Therefore, Dillingham et al. (30) presented a prediction equation for B index of the general form:

$$
\beta = 10 \t b_1 \t (C_S/C_F)^{b_2} C_F^{b_3} \xi^{b_4}
$$
 (13)

where b^1 , b^2 , b^3 , and b^a are exponents determined empirically.

The ξ index was included in the β prediction equation to account for differences in the β index when different filter aids are used. The use of the equation in this form, however, was not recommended. The ξ index is an index of the hydraulic characteristics of clean filter aid, not filtering

characteristics. For example, Dillingham and Baumann (28) found that a filter aid with a higher ξ index (K_{γ}) may possibly form a filter cake with a lower β index $(K_{\hat{4}})$, even though C_S and C_F remain the same. This was substantiated by data reported by Baumann et al. (8) and Oulman et al. (55). Dillingham felt it was best to determine separate 6 prediction equations of the form:

$$
\beta = 10 \text{ b} \cdot 1 \quad (C_{\text{S}}/C_{\text{F}})^{\text{b} \cdot 2} \quad C_{\text{F}}^{\text{b} \cdot 3} \tag{14}
$$

for each grade of filter aid used.

If the exponent b^2 is 0 , the above equation becomes:

$$
\beta = 10 \t{b} \t{10} (C_{\rm S}/C_{\rm F}) \t{b}^2 \t{(15)}
$$

which is identical to Equation 12. Dillingham included C_F (Equation 14) as a variable separate from C_S/C_F in an attempt to improve β prediction for waters containing suspended solids such as sodium montmorillonite. This improved the 6 predictions for all waters, especially for water containing sodium montmorillonite and softened water (30) , which both showed significant solids concentration effects.

Turbidity has been used as a relative measure of suspended solids concentration in studies by Regunathan (61) and Bridges (15). Regunathan used turbidity as a measure of the concentration of clay added to University tap water and Bridges measured turbidity in the field while filtering

coagulated and settled surface waters. In both cases, the use of turbidity in place of suspended solids concentration, C_S , did not reduce the effectiveness of Equations 14 and 15 as β prediction equations. This is as expected since turbidity and suspended solids concentration are directly related and a plot of turbidity versus suspended solids concentration is usually a straight line passing through the origin (1, 61). Thus:

$$
T = mcS
$$
 (16)

where :

T = turbidity, usually JTU $m = slope of T versus C_S plot$

The exponents in Equation 15, for example, are determined by taking the logarithms of both sides of the equation to give:

$$
\log \beta = b_1 + b_2 \log(C_S/C_F) \tag{17}
$$

and then performing a simple linear regression between log 6 and log(C_S/C_F). If turbidity is substituted for the suspended solids concentration in Equation 15, then:

$$
\beta = 10^{b_1} (T/C_F)^{b_2}
$$

or:

$$
\beta = 10^{b_1} (mc_S/c_F)^{b_2}
$$

and taking logarithms of both sides of the equation:

$$
\log \beta = b_1 + b_2 \log(\text{mC}_S / \text{C}_F)
$$

= $(b_1 + b_2 \log \text{m}) + b_2 \log(\text{C}_S / \text{C}_F)$ (18)

Thus, replacing the suspended solids concentration by turbidity does not change the form of the prediction equations. It acts only to change the value of the exponent b^1 . All other exponents and the regression coefficient remain the same.

Baumann et al. (5) expected that filter cakes with the same weight of suspended solids per unit weight of filter aid in the cake (C_S/C_F) would have the same resistance to flow and the same β index. It was evident, however, that filter cakes with equal C_S/C_F ratios would not have the same β index when different types or grades of filter aids were employed. This required that separate β prediction equations be determined for each filter aid that was used. Recently, Oulman and Baumann (54) suggested that the resistance to flow through a filter cake is really a function of the volume of void spaces in the cake that are available for flow and deposition of suspended solids. Thus, it was expected that a prediction equation of the form:

$$
\beta = 10^{b_1} (C_S / V_V)^{b_2}
$$
 (19)

where :

$$
V_{\nu} =
$$
 volume of voids [L³]

would have the same exponents, b_1 and b_2 , for different grades of filter aid. The volume of voids was calculated as the total volume of filter cake, assuming that the suspended solids do not increase the volume of the cake, minus the volume of the filter aid in the cake or:

 V_{V} = volume of cake - volume of filter aid

$$
=\frac{C_{\overline{F}}}{\gamma_{\overline{p}}} - \frac{C_{\overline{F}}}{\gamma_{\overline{w}} \rho_{\overline{F}}}
$$
 (20)

where :

 ρ_F = effective specific gravity of filter aid [--]

Since C_p is expressed in mg/l or ppm by weight, the volume of voids calculated by Equation 20 will be the volume of voids in a cake formed by filtering a million pounds of influent. C_S is also expressed in mg/l therefore the ratio C_S/V_V expresses the weight of suspended solids per unit of void volume in the filter cake.

Oulman and Baumann also observed from Equation 8 for the head loss through a flat filter cake that the product $\beta C_{\mathbf{F}}$ must be the same for all filter aids in order to give the same head loss under identical operating conditions. Thus, they thought it would be more reasonable to use a prediction

equation of the form:

$$
\beta C_{\rm F} = 10 \begin{bmatrix} b_1 & b_2 & b_3 \\ c_5 & c_5 & c_6 \end{bmatrix}
$$
 (21)

This equation was applied to data obtained from the filtration of University tap water containing iron floe produced by adding ferric chloride. For all filter aids used, the value of b₂ was 2 and b₃ was -1 within the precision of the data, but the value of b^1 varied for different grades of filter aid. Also, when C_S is equal to zero, βC_F must be equal to $\beta_0 C_p$ where β_0 is the filter cake resistance index for a clean filter cake. Equation 21 was modified to give :

$$
\beta C_F = \beta_0 C_F + k C_S^2 / V_V \tag{22}
$$

where :

$$
\beta_0 = \text{resistance index of a clean filter cake}
$$
\n
$$
= \xi \gamma_w 10^{-6} \quad [\text{L}^{-2}]
$$
\n
$$
k = \text{empirical constant}
$$

Further investigation revealed that k was directly related to the effective particle size of the filter aids and that a different k value-particle size relationship existed for the filter aids produced by different manufacturers. The data from which this conclusion was based are shown in Figure 11. The effective particle size, $D_{\alpha m}$, was determined from

Figure 11. Relation of k value to effective particle size

- E = Eagle-Picher Industries, Inc.
- G = Great Lakes Carbon Corp.
- J = Johns-Manville Products Corp.

 \mathcal{L}

 $S = Sil-Flo Corp.$

Figure 12. Relation of k value to hydraulic radius

permeability data and theoretical relationships derived from the Rose equation (22, pp. 302-304) which relates the resistance of flow through a porous bed to the physical properties of the material in the bed. The Rose equation was given as

$$
H_C - (\frac{1.067}{S}) (\frac{C_d}{g}) (L) (\frac{q^2}{\epsilon^4}) \Sigma_{\overline{d}}^{\underline{X}}
$$
 (23)

whe re :

$$
S = \text{particle shape factor} = 6V_p/DA_p \quad [- -]
$$
\n
$$
V_p = \text{particle volume} \quad [L^3]
$$
\n
$$
A_p = \text{particle surface area} \quad [L^2]
$$
\n
$$
C_d = \text{drag coefficient} = 24/N_R \text{ for } N_R < 10^4 \quad [- -]
$$
\n
$$
L = \text{thickness of porous bed} \quad [L]
$$
\n
$$
\varepsilon = \text{porosity} \quad [- -]
$$
\n
$$
X = \text{weight fraction of particle in a given size}
$$
\n
$$
\text{range} \quad [- -]
$$
\n
$$
d = \text{particle diameter} \quad [L]
$$

By combining Equations 8 and 23 and assuming a straight line particle size distribution and a mean uniformity coefficient, the following equation was developed for the effective filter aid particle size in microns:

$$
D_{em} = \frac{1794 (Y_w / B_0 Y_F)^{\frac{1}{2}}}{\epsilon^2}
$$
 (24)

where γ_w and γ_F have units of lb/cu ft and β_0 has units of ft^{-2} .

This equation was then used to determine an expression

for the hydraulic radius of a filter cake. From the data shown in Figure 12, Oulman and Baumann (54) found that the functional relationship between k and hydraulic radius was :

$$
k = \frac{12.0 \times 10^9}{R_{em}}
$$
 (25)

where :

R_{em} = effective hydraulic radius, in microns

Equations 8, 22, and 25 were combined to give a single relationship between the physical and filtration characteristics of all filter aids:

$$
V_{\rm m} = 3.95 \times 10^{-3} \text{ I}
$$
 (26)

where :

 V_F = volume of body feed in cu ft/MG required for the specified filtration conditions :

$$
q = 1 gpm/sq ft
$$
\n
$$
t = 6 hrs
$$
\n
$$
H_c = 100 ft of water
$$
\n
$$
C_S = 4 ppm iron
$$
\n
$$
Water temperature = 20°C
$$
\n
$$
Flat septa
$$
\n
$$
\gamma_F^{-3/2}B_0^{-1/2}
$$

 $I = filter$ aid index = ρ_{F}

It was proposed that filter aid for the removal of iron floe could then be specified as having an effective size within a particular range and a filter aid index equal to or below a stated value. It was assumed that filter aids meeting the specification produce acceptable effluent quality.

Optimum Design of Precoat Filters

The first investigations which dealt with the cost of precoat filtration were primarily concerned with the comparison of diatomite filtration to sand filtration. One of the most extensive studies was made in 1951 by Sanchis and Merrell (66) of the Los Angeles Department of Water and Power. Their purpose was to determine the applicability of diatomite filtration for the removal of taste and odor caused by plankton growths in open reservoirs. They discovered that diatomite filtration was effective in producing water of a quality comparable to that produced by conventional methods of pretreatment followed by rapid sand filtration. In addition, they found that diatomite filters had lower space requirements and lower first cost. It was concluded that for average water quality conditions, the total cost per unit volume of water was about the same for diatomite without pretreatment as for rapid sand filtration with pretreatment.

A plant scale comparison of diatomite and rapid sand filtration of Raritan River water in New Jersey was made by

Bell in 1955 (13). Three methods of filtration were used; diatomite filtration of raw water, diatomite filtration of pretreated water, and rapid sand filtration of pretreated water. The results of the investigation showed that the installation costs for diatomite filtration, with or without pretreatment, were considerably lower than for sand filtration and the operating costs for diatomite filtration were slightly higher than for sand filtration. Bell concluded that the cost of diatomite filtration of the raw water was approximately equal to the cost of sand filtration plus pretreatment.

These and similar studies helped to dispel some of the hesitancy which engineers and state health departments had to the use of precoat filters. In 1965, the American Water Works Association Task Group on Diatomite Filtration concluded that, "...diatomite filter systems, if properly designed, constructed, and operated, can be successfully used in the production of potable water for municipal use" (67). This task group also stated, "As far as the committee has discovered, no diatomite or rapid sand plant has yet been designed to operate in its most economical range, although several installations may approach this condition."

The filtration research conducted at Iowa State University has had as its ultimate goal the development of straightforward techniques which can be used to determine

the optimum design characteristics of both precoat and sand filters. The "theory of precoat filtration" was developed by LaFrenz, Dillingham, and others for use in optimizing the design and/or operation of precoat filters.

Before developing his theory of precoat filtration, LaFrenz (43) made a total of more than 120 filter runs in order to assess some of the "optimums" in precoat filtration. He made tests with three precoat filters; (1) a constant-rate pilot plant, (2) a constant-rate, bench scale filter, and (3) a constant-pressure, bench scale filter. All tests were made with University tap water to which ferrous sulfate was added. The results and conclusions drawn from these tests have been published in several articles (9, 10, 45).

Previous efforts had been made to determine the optimum amount of body feed, however LaFrenz pointed out that the optimum body feed rate depends upon which factors are used to define the optimum. He considered three optimum body feeds :

1. Filter aid economy optimum body feed - That body feed which produces the maximum number of gallons of potable filtrate per pound of filter aid for a given water, filter, and type and grade of filter aid.

2. Head loss optimum body feed - That body feed which produces the maximum number of gallons of filtrate per filter

run, when filtering to some specific head loss. It is also the body feed which will produce the lowest head loss for the production of a specific amount of filtrate.

3. Overall optimum body feed - That body feed which produces potable water at the minimum cost per gallon.

These three optimum body feeds are not the same even under identical filtering conditions. This fact was clearly shown by LaFrenz's data in Figure 13. For this data, the head loss optimum body feed is approximately 600 ppm whereas the filter aid economy optimum body feed is only 60 ppm.

At first glance, one might expect that the head loss optimum body feed would be infinite since as more body feed is added, the more porous the filter cake becomes, and therefore the run length increases. Eventually, however, the point is reached where the beneficial effects of increased porosity are balanced by the detrimental effects of the increased thickness of the filter cake. At higher body feed rates, this detrimental effect outweighs the beneficial effect, shortens the run length and decreases the total volume of filtrate during the run. The body feed rate at which these effects are balanced is the head loss optimum body feed. It varies only with factors which determine the composition of the filter cake such as type and grade of filter aid and type and amount of suspended solids.

Figure 13. A comparison of diatomite economy and head loss optimum body feed

> LaFrenz's (43) data with constant-rate, bench scale filter with:

$$
C_S = 7-8 ppm iron\nq = 1 gpm/sq ft\nH_t = 20 ft\nw = 0.20 lb/sq ft
$$

The filter aid economy optimum body feed was found to depend on both the amount of precoat and the terminal head loss. LaFrenz found that, in general, the filter aid economy optimum body feed decreased when the amount of precoat was decreased and the terminal head loss increased.

In designing a filtration plant, the engineer's principal concern should be to design a plant which can be operated to produce the desired amount of potable water at the least possible cost per gallon. Therefore, the optimum body feed which he should evaluate is the overall optimum body feed. Baumann and Babbitt (4) found that the most important factors affecting the cost of precoat filtration are: raw water quality, flow rate, terminal head loss, and the type, grade, and amount of filter aid. Thus, for a particular source of raw water and filter aid, there is a set combination of body feed rate, filtration rate, and terminal head loss which together result in the production of water at the least cost per gallon. This combination is the optimum body feed, optimum filtration rate, and optimum head loss. The optimum body feed is by definition the overall optimum body feed.

LaFrenz and Baumann (45) noted that the above three optimums are influenced by four basic cost factors: filter aid, labor, power, and equipment. They then presented a procedure for manually calculating the optimum combination of body feed, filtration rate, and terminal head loss. Before

this procedure can be used, the type of filter and the type and grade of filter aid must be chosen. LaFrenz (43) stated that both cylindrical and flat septums are acceptable and that the most economical filter aid is the coarsest grade which will produce an acceptable quality of water. Basically, his procedure consists of choosing a filtration rate and body feed rate and then calculating the total cost per 1000 gal at various terminal head losses. From these calculations, the minimum total cost at the chosen filtration rate and body feed rate can be obtained (Figure 14). Similar calculations are made at several body feed rates for each filtration rate that is considered. The minimum total costs obtained from all of these calculations are then plotted against the rate of body feed as is done in Figure 15. From this figure, the minimum total cost and optimum design conditions can be obtained.

To use LaFrenz's procedure for calculating optimum precoat filter design, several filter runs must be made. The data from these filter runs are needed in order to define the length of run for each combination of body feed, filtration rate, and terminal head loss. In addition, this procedure requires many time consuming calculations. ' Therefore, Dillingham (25, 27) developed a digital computer program named POPO (Program for Optimization of Plant Operation) which can be used to design a precoat filtration plant which will operate at least cost. Minimum, maximum.

Figure 14. Effect of terminal head loss on filtration costs

> LaFrenz's (43) data with constant-rate, bench scale filter with;

 $C_S = 7-8$ ppm iron $q = 1$ gpm/sq ft $w = 0.15$ lb/sq ft

Figure 15. Total minimum cost as a function of body feed for different rates

> LaFrenz's (43) data with constant-rate, bench scale filter with:

 $C_S = 7-8$ ppm iron $w = 0.15$ lb/sq ft

 ~ 10

 $\mathcal{L}^{(1)}$

 \sim

 $\ddot{}$

and incremental values of body feed rate, filtration rate, and terminal head loss are read into the computer. POPO then determines and prints out the ten most economical combinations of these three factors and the respective filtration costs. The program employs a β prediction equation (Equation 13) to calculate B for each body feed rate and uses Dillingham's precoat filtration equations to determine the length of run for each combination of body feed rate, filtration rate, and terminal head loss. Thus, the only filter run data needed are those necessary to define adequately the B prediction equation.

Dillingham used POPO and data from previous research at Iowa State University to optimize the design of several hypothetical installations. He also collected data at a lime-soda ash softening plant at Lompoc, California, and used POPO to optimize the operation of that precoat filtration plant. His conclusions from this work were:

1. Cylindrical septa are more economical than flat septa. The smaller the diameter of cylindrical septa, within practical limits, the greater the economy.

2. A smaller grained filter aid may prove to be more economical than a coarser grained filter aid, even though the smaller grained filter aid results in greater head loss at the same body feed rate. This is because smaller grained filter aids generally cost less per unit weight.

These conclusions invalidated LaFrenz's suggestions that either cylindrical or flat septa are acceptable and that the most economical filter aid is the coarsest grade which will produce acceptable water.

PURPOSE AND SCOPE

The development of the theory of precoat filtration and the techniques for optimizing the design of precoat filters have been reported in numerous theses, reports, and published papers over the past fifteen years. As previously stated, the primary purpose of this thesis is to present the method for optimum design of precoat filters in a form which is readily usable to the design engineer. An additional objective is to review the present theory of precoat filtration in order to determine its shortcomings and to point out its limitations. In essence, this thesis represents an up-to-date design manual for precoat filters. The scope of this thesis is limited to the theory and procedures for determining the conditions that an engineer must specify for the optimum design and operation of precoat filtration plants. Specific details concerning the construction of various filter components and general considerations in the design of any water treatment facility will not be included.

The problems that an engineer encounters in the design of a precoat filtration plant may be classified in three broad categories. These are data collection, data reduction, and determination of optimum design conditions. The specific objectives of this thesis were established to answer the

questions that an engineer might have in each of these areas. The specific objectives and the categories in which they are contained are:

Data Collection

- 1. To explain what data are needed, what important variables are involved, and over what range of these variables the data should be collected. This includes both cost data and raw filtration data obtained by pilot plant studies at the proposed plant site.
- 2. To show how data can be collected without elaborate pilot plant equipment to determine the filtering characteristics of the raw water.

Data Reduction

- 3. To explain how to calculate the filter cake resistance index or 3 index from the filtration data and to demonstrate the effects of certain factors on the value of the 6 index.
- 4. To show how to develop B index prediction equations and to provide possible insight into the form of the resulting equations.

Determination of Optimum Design Conditions

- 5. To demonstrate how the optimum design conditions may be obtained.
- 6. To show how several of the filtration variables

and cost factors influence the optimum design conditions.

In past research projects concerned with precoat filtration, considerable reliance has been placed in the use of the digital computer. Manuals for computer programs which have been developed for use in determining β indices, β predictions equations, and optimum design conditions will be presented in this thesis. It is not intended however that the computer and a knowledge of computer programming be a necessity for the design of precoat filters. Therefore, an additional objective of this thesis is to present and demonstrate procedures for manually calculating β indices, β prediction equations, and optimum design conditions.

The development of a sound, rational theory is probably the most important prerequisite to the optimum design of any process. Therefore, a review was made of the present theory of precoat filtration in order to determine in what respects it might be improved. In past research, two factors concerning the theory of precoat filtration have caused the most problems. First, the β index prediction equations have been developed empirically and no attempt has been made to derive prediction equations on a rational basis. Thus, the β prediction equations that have been developed for various waters are useful only within the range of data on which

the equations are based and do not account for changes in variables that remained constant during the collection of the original data. The second factor which has caused problems is initial dilution. In many instances it has been found that the theoretical dilution rate does not adequately account for effects attributed to initial dilution. It has been assumed that this is due to inadequate mixing or shortcircuiting within the filter housing. Therefore, to determine the β index from filtration data, the effects of initial dilution are generally either neglected or else the initial dilution rate is selected to fit the raw data. In the determination of optimum design conditions, initial dilution effects have been completely ignored. What effect this may have has never been examined. Therefore, a further objective of this thesis is to study the period of initial dilution and to suggest how its effects may be included in the determination of optimum design conditions.

DATA COLLECTION

Filtration Data

General

In the design of a precoat filter plant for a municipal water supply, the first consideration should be the determination of the characteristics of the raw water which will fix the plant design. The characteristics which must be determined are :

- 1. Temperature of the raw water.
- 2. Suspended solids concentration in the raw water.
- 3. Variation of the β index with changes in the body feed concentration and suspended solids concentration.

These characteristics are of primary importance because they determine the combination of filtration rate, body feed concentration, and terminal head loss required for optimum design. In addition, it is necessary to determine the most economical grade of filter aid to be used to obtain the desired filtered water quality.

Design water quality

The temperature of and concentration of suspended solids in some waters, in particular those from ground water sources, may remain relatively constant. Other waters, such as a river water, may have large variations in both temperature and quality. If this is the case, it is necessary to analyze

variations in water quality which have occurred over a long period of time in order to determine the temperature and suspended solids concentration (and their filtration characteristics) to be used in design. This temperature and suspended solids concentration would be used to determine the optimum combination of filtration rate, terminal head loss, and body feed rate. Once the filtration rate and terminal head loss are set, the optimum body feed rate can be calculated for the temperature and turbidity variations which are to be expected as a guide for the operation of the filter.

River waters in northern climates have an annual variation in both temperature and turbidity. For example, graphs showing the variation in temperature and turbidity of weekly samples collected during 1968 and 1969 from the Des Moines River near Boone, Iowa, are shown in Figures 16 and 18 respectively. Frequency distribution diagrams for these 104 weeks of data are presented in Figures 17 and 19- During 1968 the mean temperature of the water was 12.9 °C and during 1969 it was 11.5 °C. The mean turbidity was 30.5 JTU during 19 68 and 29.6 JTU during 1969. If a precoat filter were to be used to filter this water, the design temperature and turbidity must be chosen to achieve the minimum annual cost of filtration. This will involve a study of how the annual cost of filtration is affected by the design conditions of the filter. An example showing how the design water quality affects the annual cost of filtration will be

Figure 16. Variation of water temperature observed in the Des Moines River at Boone, Iowa

Figure 17. Frequency distribution diagram of temperatures observed in the Des Moines River at Boone, Iowa

Number of occurrences in 104 (weekly) samples.

 $\overline{48}$

30

TEMPERATURE, ⁰C

 σ_{0}^{L}

Figure 18. Variation of water turbidity observed in the Des Moines River at Boone, Iowa

Figure 19. Frequency¹ distribution diagram of turbidities observed in the Des Moines River at Boone, Iowa

 ~ 400

 1 Number of occurrences in 104 (weekly) samples.

presented in a later chapter (p. 188).

g prediction data

General The relative value of the β index and its variation with changes in the concentrations of body feed and suspended solids are characteristic of the water filtered. Past research indicates that the value of the 8 index is greatly influenced by the type of suspended solid. Hawley (37) filtered University tap water to which ferric chloride and ferrous chloride were added. Celite $535¹$ was used as the filter media in both cases. His data showed that for the same iron concentration and body feed rate, the β index for water containing ferric iron floe is almost 25 times that for water containing ferrous iron floe. Laboratory data gathered by Regunathan (61) also indicated large differences in the characteristics of waters containing Kentucky Ball $clay^2$ and Wyoming bentonite $clay³$ when both waters were filtered using Hyflo Super-Cel⁴ as the filter media. At equal ratios of influent turbidity to body feed rate, the β index for water

 1 Diatomaceous earth filter aid, Johns-Manville Corporation, New York, N.Y.

²Old Hickory No. 5 Ball Clay, Old Hickory Clay Co., Paducah, Kentucky.

³ Black Hills Bentonite, International Mineral and Chemical Corporation, Skokie, Illinois.

⁴ Diatomacepus earth filter aid, Johns-Manville Corporation, New York, N.Y.

containing Wyoming Bentonite clay was over 70 times that observed for water to which Kentucky Ball clay had been added.

The large range of values of the β index indicated by these results demonstrates that the only way to determine the value of the β index and to describe its variation with body feed and suspended solids concentrations is by collecting filtration data with the water to be filtered at the proposed plant site. Even data collected at a nearby plant which filters water from a similar source may be of little value. For example, Creston, Iowa, and Albia, Iowa, are both located in the central part of southern Iowa and obtain their water supplies from impounding reservoirs. A pilot filter was used to gather data on these waters after they had been treated by coagulation using alum and lime followed by settling (15) . It was found that at the same ratio of turbidity to body feed rate, the 6 index for the water at Albia was approximately 30 times that for the Creston water.

The data necessary for the prediction of the β index at various suspended solids concentrations and body feed rates can be collected by making a-few filter runs with a pilot filter. A bench-scale filter which was designed for this purpose will be described later in this section. The minimum data collected during a filter run should include:

1. Filter aid grade.

- 2. Amount of precoat.
- 3. Filtration rate.
- 4. Water temperature.
- 5. Body feed rate,
- 6. Influent suspended solids concentration.
- 7. Effluent suspended solids concentration.
- 8. Head loss across the filter at regular intervals of elapsed filtration time.

Filter aid grade The main requirement of the filter aid is that an effluent of acceptable quality be produced. A few filter runs using different filter aid grades can be made to determine the coarsest grade that will still produce an acceptable effluent quality. The recommended procedure is to start with the coarser grade of filter aid and to use progressively finer grades in successive test runs until a filter aid is found which will produce consistently the desired water quality. The selection of the most economical grade of filter aid from those which produce acceptable effluent is complicated by the fact that the coarser grades of filter aid cost more per unit weight. Thus, switching to a coarser grade of filter aid may reduce the costs of labor, precoat filter aid, and backwashing due to an increase in run length but still result in higher overall operating costs. As example, data presented by Baumann et al. (8) were used to

prepare Table 5. The filter aids in Table 5 are arranged from coarse to fine according to the body feed rate required to give equivalent performance under identical filtration conditions. The filtration conditions are the filtration of water containing 4 mg/1 of iron at a temperature of 68 °F. Equivalent performance is defined such that filtering with flat septa at a filtration rate of 1 gpm/sq ft for 6 hours

Filter aid	Unit cost оf filter aid, $\frac{c}{1}$	Equivalence performance		Optimum performance
designation		ppm $C_{\rm F}$,	Filter aid $cost, \frac{5}{MG}$	Total cost, $\frac{2}{2}$ /MG
S ₂	4.365	76	27.63	78.5
S3	3.815	104	33.37	78.8
S ₄	3.715	112	34.66	79.9
J4	5.000	114	47.48	88.0
J3	4.900	118	48.16	88.2
E6	5.000	125	52.06	93.4
JO	4.650	141	54.62	92.6
G4	4.900	162	66.12	94.9
E5	4.900	178	72.65	97.3
E2	4.750	189	74.78	99.1
Gl	4.750	227	89.82	98.1

Table 5. Filtration costs using different filter aids

results in a head loss through the filter cake of 100 feet of water. It is seen that the cost of body feed per million gallons of filtrate increases with finer filter aids under

these circumstances. All other costs, except precoating, will be equal for different filter aids since the run lengths and terminal head losses are identical.

The overall optimum filtration costs (Table 5, last column) were determined for the same filtration conditions ($C_S=$ 4 mg/l iron and a temperature of 68 °F) using the computer program POPO. Cost data gathered by Dillingham (25, 27) and the following design conditions were used in this analysis :

- 1. Design flow = 1 MGD
- 2. Salvage value = 15 percent of first cost
- 3. Energy conversion = 70 percent
- 4. Interest rate = 4 percent
- 5. Plant life = 25 years
- 6. Precoat weight = 0.15 lb/sq ft
- 7. Power cost = 2 cents/kwh
- 8. Flat septa
- 9. Backwashing requires 10 gal of water per sq ft of filter area
- 10. 30 min are required for precoating and backwashing during each filter run

From the results of these analyses it appears that there is little difference in the total cost per million gallons when different filter aids are used. In general, the coarser filter aids are cheaper to use. However, if there is a large difference in price between two filter aids, it may be necessary to determine β prediction equations and optimum costs for both filter aids. In this example, only two filter aids finer than those higher on the list in Table 5, JO and Gl, are cheaper to use than a coarser grade for filtering iron floe.

Precoat weight and filtration rate The weight of precoat filter aid per unit septum area and the filtration rate used in collecting filtration data should be within the range of values used in practice (approximately 0.05 to 0.15 Ib/sg ft and 0.5 to 2.0 gpm/sg ft, respectively). A precoat weight of 0.15 Ib/sq ft and a filtration rate of 1.0 gpm/sg ft have been most commonly used for data collection in past research projects.

Water temperature The temperature measured should be that of the water as it passes through the filter cake. A temperature increase as high as 6 °C has been observed for water as it passed through a bench scale filter. If possible, the temperature of the water within the filter housing should be measured, if not the effluent temperature should be recorded. In any event, the water temperature should remain constant during a filter run and precautions should be taken to prevent heating of the raw water when it passes through the filter pump.

Body feed (C_F^-) and suspended solids (C_S^-) For a particular filter aid and suspended solid, the body feed rate and suspended solids concentration determine the composition and characteristics of the filter cake. Therefore, C_F and C_S are the only variables necessary to predict the β index under these conditions. The data necessary to develop a 6 prediction equation can be obtained from a few filter runs at different values of $C_{\rm g}$ and $C_{\rm F}$. The precoat weight, filtration rate, and temperature may be the same for each filter run.

Two important considerations must be kept in mind when collecting data for different combinations of C_p and C_S in order to develop a 6 prediction equation. First, the theory of precoat filtration was derived under the restriction that the body feed rate must be high enough to form an essentially incompressible filter cake. Any attempt to predict the B index for values of C_F and C_S which do not result in the formation of an incompressible filter cake is meaningless. Therefore, during the process of collecting data for β index prediction, it is imperative that the point where the filter cake becomes compressible be determined. This can be done by finding what the highest ratio of C_S to C_F is before a plot of head loss versus time becomes exponential.

The second important point to remember is that in order to determine valid regression coefficients, data must be

gathered over a significant range of values for each variable included in the 3 prediction equation. Much of the laboratory data for the filtration of iron floe have been collected with C_S held essentially constant and only C_F varied. These data were then used to develop ß prediction equations of the form:

$$
\beta = 10^{b_1} (C_S/C_F)^{b_2} C_F^{b_3}
$$
 (14)

and

$$
\beta = 10^{b_1} (C_S/C_F)^{b_2}
$$
 (15)

Since C_S was held constant, a β prediction equation such as:

$$
\beta = 10^{b_1} \, c_F^{b_2} \tag{27}
$$

would be equally valid, however it could only be used to predict β for an influent with a suspended solids concentration equal to that used to collect the data from which the equation was developed. A prediction equation with the form of Equation 15 developed from such data can be used to predict β for an influent with a different suspended solids concentration only if the assumption is made that the β index is the same for equal ratios of C_g/C_F .

If C_S is held constant, then Equation 14 can be written as :

$$
\beta = (10^{b_1} \text{ C}_S^{b_2}) (1/\text{C}_F)^{b_2} \text{ C}_F^{b_3}
$$
\n
$$
= 10^{b_1} \text{ C}_S^{b_2} (1/\text{C}_F)^{b_2 - b_3}
$$
\n
$$
= (10^{b_1} \text{ C}_S^{b_3}) (\text{C}_S/\text{C}_F)^{b_2 - b_3}
$$
\n
$$
= 10^{a_1} (\text{C}_S/\text{C}_F)^{a_2}
$$
\n(28)

where :

 $10^{a_1} = 10^{b_1}$ $c_3 = a$ constant

$$
a_2 = b_2 - b_3
$$

Thus, when only C_{p} is varied, b_{2} and b_{3} are not coefficients of independent variables and even with filtration data gathered under identical conditions, their individual values may vary widely as long as the value of b_2-b_3 remains the same. A good example of this is demonstrated by the β prediction equations presented by Baumann et al. (8) in Table 6. Note that these prediction equations were developed from data collected with C_S held constant at about 8.0 mg/1 (7.5 to 8.5 range) of iron. The coefficients b₂ and b₃ vary considerably, even for the same grade of filter aid. A few calculations can be made to show that b_2-b_3 of Equation 14 is approximately equal to $b₂$ of Equation 15 and that 10^{b1} C_S^b3 (where C_S = 8 mg/1) of Equation 14 is approximately $b,$ equal to 10 $^{\frac{1}{2}}$ of Equation 15 as suggested by consideration of Equation 28. The prediction equations developed in the

Table 6. g prediction equations presented by Baumann e;t (8)

المناجات والمحا \sim masses a component \sim **Constants : Source of iron: FeCl_ solution in tap water Effluent Iron Concentration: O.to 0-06 mg/X** plus 0.5 mg/*t* of copper sulfate, as copper.
Temperature: 60°F **Cg: about 8.0 mg/£ (7.5-8-5 range) S.: 1 gpm/sq ft Cp: variable between runs Filter area: 3.20 sq ft Çs/<^: ranges frcra about 0.02 to 0.2 between runs**

 $\ddot{}$

form of Equation 14 are valid only when the value of c_{S} is the same as that from which the equations were developed. If this is not realized, considerable error will result. For example, consider the Equation 14 prediction equations developed for filter aids S4-1 and S4-2. These are two bags of the same grade of filter aid, thus one would not expect a very great difference between the values of β calculated with the same values of C_S and C_F . If $C_S = 8$ mg/l (which is the value of C_S at which the filtration data were gathered) and $C^F = 160$ mg/1 so that C^C _F = 0.05, then:

 β for S4-1 = 42.95 x 10⁶ ft⁻²

and

 β for S4-2 = 49.52 x 10⁶ ft⁻²

which differ by only 14 percent. However, if $C_S = 4$ mg/1 and $C_F = 80$ mg/l so that C_S/C_F is still 0.05, the calculated β indices are:

 β for S4-1 = 12.84 x 10⁴ ft⁻²

and

 S for $S4-2 = 29.34 \times 10^{8}$ ft⁻²

which differ over 20,000 fold!

Any measurement which is directly proportional to the concentration of suspended solid in the filter influent may be used in place of C_S for predicting β indices. In laboratory projects, iron concentration has been used as a

measure of iron floe concentration and, in field and laboratory studies, turbidity has been found to be an acceptable measure of C_{c} .

Effluent suspended solids The concentration of suspended solids (or turbidity, etc.) in the filter effluent should be measured occasionally during each filter run to assure that the filter is working properly. An imperfection in the filter septum or an improperly formed precoat may be detected in this way. These measurements are also useful for predicting the quality of water which would be obtained by large scale precoat filtration.

Head loss The head loss across the filter cake should be measured at uniform time intervals during the filter run and it is recommended that a plot of head loss versus elapsed time of filtration be made as the filter run progresses. Any unusual changes in the body feed rate, suspended solids concentration, filtration rate, etc., can be detected by a change in the slope of this curve. The filter run should be continued until a well-defined curve is obtained. In past studies, filter runs have usually been extended one to two hours past the period of initial dilution and head loss measurements made every 10 to 30 minutes.

Bench scale apparatus

Background The need for a small, inexpensive, easy-

to-operate apparatus for collecting filtration data with the water at the proposed plant site was recognized as early as 1961. LaFrenz (43) built and operated a bench scale, constant rate filter with which he gathered the data used to evaluate the optimums in precoat filtration. LaFrenz concluded, however, that it was impossible to correlate the results from the bench-scale filter to those from a largescale pilot plant. This conclusion was based on comparison of the head loss curves obtained with these filters. The differences in the results observed for the two filters were thought to be due to differences in the ratio of septum area to volume of filter housing (initial dilution effects) and the shape of septums (increasing area effects).

In the time since LaFrenz's work was completed, the effects to which he attributed the difference in head loss between model and prototype filters have been accounted for in the equations for precoat filtration derived by Dillingham (27). A true comparison of the model and prototype results would be to compare values of the β index for filter runs which should have formed identical filter cakes; i.e. equal body feed rates and suspended solids concentrations. β indices for all of LaFrenz's filter runs with both the pilot plant and bench-scale filter were calculated and are included with the summary of filter runs in Appendix A (Tables 22 and 23). In Figure 20, a comparison of β indices at equal values of C_S/C_F is made for all of LaFrenz's filter runs. There is

Figure 20. Comparison of LaFrenz's (43) data for pilot plant and bench-scale filters PP-x (C_S,q) = pilot plant data VHP-x (C_c,q) = bench-scale filter (Variable Head Permeameter) data X = series number

considerable scatter in the results obtained with the benchscale filter. However, these results do cluster around the results of the filter runs made with the pilot plant. Some of the scatter in the results for the bench-scale filter may possibly be due to differences in iron concentration or filtration rate. If only the bench-scale filter results for runs with the same iron concentration and filtration rate as used during the pilot plant runs (8 mg/1 iron and 1.0 gpm/sq ft) are used in the comparison (see Figure 21), the result is still the same. The bench-scale filter results shown in Figure 21 vary from the pilot plant results (at equal C_c and C_p) by as much as -50 percent to as little as +10 percent. It appears then, that contrary to LaFrenz's conclusion, a bench-scale, constant-rate filter may be used to predict the results of a full-scale filter.

During 1967 and 1968, research was conducted at Iowa State University in order to develop specifications for filter aids used by the U.S. Army (11). During the initial stages of this project, two bench-scale filters were built for determining the ξ index and in-place bulk density of filter aids, a constant-pressure filter similar to that used by Al-Khafaji (la) and a constant-rate filter. Because of its simplicity of operation and the short time required to make a filter run, the constant-pressure filter was used exclusively throughout the study.

Figure 21. Comparison of LaFrenz's (43) data gathered under identical filtration conditions $(C_g = 8 mg/l iron, q = 1 gpm/sq ft)$

PP-x = pilot plant data

VHP-x = bench-scale filter (Variable Head Permeameter) data

X = series number

An investigation of the possibility of the use of the constant-pressure filter to predict results for constantrate filtration was recently completed by Arora (3). He concluded that constant-pressure results could not be used to predict constant-rate results, the primary reason being the inherent differences in the porosity distribution of the cakes laid in the two processes. The constant-rate filter built during the early stages of the filter aid specification study (11) was then evaluated for use in predicting the results of large-scale, constant-rate filtration. Arora found that this apparatus was successful in predicting the filter cake resistance obtained in filter runs made using the pilot plant and iron-bearing waters (8). The apparatus and its operation are described in the following sections.

Description The apparatus is called the small-scale, constant-rate filter or SSCR filter. A photograph of it is shown in Figure 22. The unique feature of this apparatus is the system of three gears and three 3-way plug valves which facilitates switching from precoating to filtering and filtering to backwashing cycles by turning a single lever. A photograph of this valve system is shown in Figure 23 and working drawings are contained in Appendix C. Other components of the apparatus include:

1. A precoat pot of about one liter capacity and made out of plexiglas. Working drawings are given in

Figure 22. The SSCR filter apparatus

- 1 = Precoat pot
- 2 = Filter cell
- $3 =$ Pump
- $4 =$ Rotameter
- 5 = Raw water and backwash water holder
- $6 =$ Manometer
- B = Backwash position
- $F =$ Filter position
- $P = Precoat$ position

Figure 23. View from behind control valves (top plate removed)

- $1 =$ Pump inlet
- 2 = Rotameter inlet
- 3 = Filter outlet
- 4 = Pump outlet
- 5 = Precoat inlet with precoat control valve
- 6 = Precoat outlet
- 7 = Filter inlet
- 8 = Backwash waste

Appendix C.

- 2. A raw water and backwash water holder with a capacity of two liters, made of plexiglas. Working drawings are given in Appendix C.
- 3. Pump Model No. 2, E-38N, Patent No. 194,570. Little Giant Pump Co., Oklahoma City, Oklahoma.
- 4. A filter assembly built of plexiglas to permit viewing of the precoat and filter cake. The flat filter septum is two inches in diameter (3.142 sq in.). Working drawings are contained in Appendix C.
- 5. Mercury manometer Model BUB-24, 64 cm. King Engineering Corp., Ann Arbor, Michigan.
- 6. Rotameter Model No. 2-1355-V, SHO-RATE. Brooks Instrument Co., Inc. Hatfield, Pa. A tube of size R-2-15-C is used with a 1/8-inch diameter stainless steel float. Maximum flow capacity with this arrangement is about 320 ml/min which corresponds to a filtration rate of 3.9 gpm/sq ft through a two inch diameter filter.
- 7. Magnetic stirrers (Magnestir, Catalog No. 52617, Chicago Apparatus Company. Chicago, Illinois) with two inch stirring bars are used to prevent settling or segregation of the contents of the precoat pot and raw water holder.

A schematic diagram of the SSCR filter is shown in

Figure 24. All connections between filter components are made with 1/4-inch ID tygon tubing. A needle valve at the inlet of the rotameter tube is used to regulate the filtration rate and a gate valve on the inlet line to the precoat pot is provided to control the flow rate during precoating. To facilitate removing air from the filter system and to prevent air binding of the pump, the apparatus components should be arranged so that the pump is at the lowest level followed by the control valves, filter cell, raw water holder, and then the manometer inlets and air bleeds at the highest level.

Operation Prior to the start of a filter run, the following materials should be available:

- 1. At least 10 liters (approximately 4 gallons) of raw water.
- 2. Approximately 5 gallons of clean water (preferably distilled) for precoating and backwashing.
- 3. A weighed amount of precoat. For a precoat of 0.15 Ib/sq ft on the 2 inch diameter filter, 1.5 grams of filter aid are required.
- 4. 8-10 weighed amounts (+0.0002 gram) of the filter aid required per liter of raw water.

Instructions for making a complete filter run with the SSCR filter are outlined below:

Figure 24. Schematic diagram of the SSCR filter apparatus

- b. indicates flow path during filtering
- c. indicates flow path during backwashing

 $\left\{\right.$ shown by thick lines

J

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- $1.$ Fill both the precoat pot and raw water holder with clean water.
- $2.$ With the control lever in filter position, start the pump.
- Open the air bleed lines to remove air trapped on $3.$ both sides of the filter septum. If air bubbles become bound in the pump impeller, stop the pump and let the trapped air move into the pump effluent line.
- 4. After all of the air has been removed from the pump, filter cell, rotameter, and manometer lines, turn the control lever to precoat position and open the precoat control valve wide open. Most of the air trapped in the precoat lines will be removed to the precoat pot, however some may become trapped in the pump or filter cell.
- $5.$ Turn the control lever to backwash position. Any air now trapped in the pump or filter cell can be removed by turning the control lever to filter position and opening the air bleed lines.
- 6. Repeat steps 3, 4, and 5 if necessary.
- Set the precoat control valve to provide the proper $7.$ flow rate for precoating. Too low a flow rate allows settling of filter aid in the filter cell and too high a flow rate causes an uneven precoat layer to

form. The proper rate was found to be with the precoat control valve open 2-1/2 turns and a 4 cm Hg head loss at the end of the precoating step (using 0.15 Ib/sq ft precoat).

- 8. With the control lever in filter position, add the precoat filter aid to the precoat pot and turn on the magnetic stirrer under the precoat pot.
- 9. Turn the control lever to precoat position.
- 10. While the filter is being precoated, remove any clean water that remains in the raw water holder. After the water in the precoat pot has become clear and a uniform precoat layer has formed, fill the raw water holder with one liter of the water to be filtered and a weighed amount of body feed filter aid. Turn on the magnetic stirrer under the raw water holder and turn off the one under the precoat pot.
- 11. Simultaneously turn the control lever to filter position and start the stopwatch or note the clock time.
- 12. Immediately adjust the rotameter needle valve to give the desired filtration rate. The flow rate needed to give a filtration rate of 1.0 gpm/sq ft through a 2-inch diameter filter is 82.6 ml/min. This corresponded to a reading of 26 with the

rotameter arrangement described previously.

- 13. Observe and record the head loss and elapsed filtration time at appropriate intervals. Plot the head loss versus time curve as the run progresses. Also record the effluent temperature and effluent quality at various times during the filter run. At least once during the run, check the flow rate by collecting the effluent over a period of 10 minutes in a graduated cylinder. The rotameter should be watched carefully to be sure that a constant flow rate is maintained.
- 14. Add an additional liter of raw water and body feed when needed. This is necessary approximately every 10 minutes for a 2 inch diameter filter and 1.0 gpm/sq ft filtration rate.
- 15. At the end of the filtering cycle, allow the raw water holder to empty. Then fill the holder with clean water.
- 16. Open the rotameter needle valve so that any filter aid particles that may have lodged in the valves are removed on the filter cake. Keep the raw water holder full of clean water.
- 17. Turn the control lever to backwash position. It may be necessary to alternately turn the control lever from backwash to filter positions in order to

break up large pieces of filter cake.

18. Another filter run can now be made starting with Step 8. To assure that there is no air in the system, begin with Step 3.

Cost Data

General

The collection of accurate cost data is as important for the optimum design and operation of a precoat filtration plant as the collection of accurate filtration data. Costs vary greatly from one location to another and over a period of time. Therefore, it is important that the costs used in design be appropriate for the location of the proposed plant and the time it is to be built. If any of the cost factors used in the design of a plant change greatly after the plant is in operation, the operating conditions can be changed to achieve minimum operating cost under the new conditions.

The total cost of filtration is composed of the first cost of the plant and its operating costs. First cost represents the costs of the building, land, filters, pumps, body feeding equipment, piping, etc. Operating costs include the costs for filter aid, power, labor and maintenance. Other costs connected with the administration of a water supply system generally do not vary with the choice of design conditions and therefore are not included in the determination

Ill

of optimum design conditions.

The cost data needed are discussed in the following sections. The used of these data in determining the optimum design conditions will be discussed in a later chapter.

First cost

Once the type of building construction and plant location are chosen, the first cost of a plant is a function of the filter area to be provided and the filtration rate. The filtration rate is a factor because, for a specific filter area, the filtration rate determines the size of pumps, piping, body feeders, and other equipment which vary in size and cost according to the total quantity of flow. Therefore, the first cost data needed is that to define a curve of first cost in \$/sg ft versus filter area for a particular filtration rate and a filtration rate factor. The filtration rate factor is defined as the percent increase in first cost per 1,0 gpm/sq ft increase in the filtration rate. Enough first cost-area data must be available to allow linear interpolation between points with little error.

Additional information required for amortizing the first cost are the salvage value in percent of the first cost, the plant life, and the annual interest rate.

Operating costs

The only cost data necessary for calculating the cost

of filter aid is the price per ton of filter aid delivered to the plant site.

Power costs are determined by the volume of water filtered and the total head pumped against according to the equation :

$$
CP = \frac{QH_t}{E} \left(\frac{8.34 \text{ lb/gal}}{2.655 \text{ x } 10^6 \text{ ft-lb/kwh}} \right) \quad C \tag{29}
$$

where :

 $CP = \text{cost of power in } \frac{5}{\text{unit time or } \frac{5}{1000}}$ gal $Q =$ volume of water filtered in gal/unit time or equals 1000 gal depending on desired cost basis H_+ = terminal head loss in ft E = overall efficiency of energy conversion $C = energy cost in $\frac{6}{x}$$

Thus, the cost data necessary for computing the power cost for pumping are the unit cost per kilowatt-hour and the overall efficiency of energy conversion.

It is assumed that the cost of labor and maintenance depend on the size of the plant just as first cost. Therefore, the necessary data are the cost of labor and maintenance in \$/sq ft versus filter area for a particular filtration rate and the filtration rate factor.

The need to backwash and reprecoat a filter at the end of a filter run affects filtration costs in two ways:

1. Since no filtered water can be produced during the backwashing cycle, the filter area must be increased so that the water produced during the filtering cycle will be enough to provide the required plant output each day in spite of the filter down time for backwashing.

2. Since filtered water is used in backwashing and wasted, the filter area must also be increased slightly to provide daily the water needed for backwashing during the normal filtering cycle.

The data necessary to calculate the increased filter area required are the volume of clean water in gal/sq ft required for backwashing and the length of time per filter run needed to backwash and precoat the filter.

DATA REDUCTION

g Index Determination

Flat septum

The head loss through a flat filter cake is determined by the formula:

$$
H_{\mathbf{C}} = \sigma X \tag{8}
$$

where :

$$
\sigma = q^2 v \beta C_{\rm F}/g
$$

and

$$
X = t - (1 - e^{-\delta t}) / \delta
$$

If the head loss, H_c , is plotted versus X, the resulting curve should be linear with slope σ . Thus, by determining the value of the slope, the value of β contained in σ can be calculated since all other components of a are known for a particular filter run.

It has been found that it is difficult to determine an exact value of the theoretical dilution rate, 6, primarily due to a lack of complete mixing within the filter housing. However, since:

 $\lim_{t \to \infty} (1-e^{-\delta t})/ \delta = 1/\delta$ $t\rightarrow\infty$

then for large values of t:

$$
x = t - 1/\delta
$$

= t - t_d (30)

where :

 t_a = theoretical detention time, V_f/Q

This shows that the effect of initial dilution is to offset the head loss versus time curve by a length of time equal to $t^{\mathcal{A}}$ (Figure 25). Thus, to determine the value of B from data gathered using a flat filter septum, simply measure the slope of the linear portion of the head loss versus time curve and calculate β as:

$$
\beta = \frac{q}{q^2 v} \frac{\text{slope}}{C_F} \tag{31}
$$

Cylindrical septum

The head loss through a cylindrical filter cake is determined by the formula:

$$
H_C = \frac{R_S \sigma}{\phi} \ln(1 + \frac{R_S \phi X}{R_O})
$$
 (6)

where :

$$
\phi = 2q\gamma_{w}C_{F}(10^{-6})/\gamma_{p}
$$

If the head loss, H_c , is plotted versus $\ln(1 + R_s \phi X/R_0^2)$, the resulting curve should be linear with slope $R_{\alpha} \sigma/\phi$. By determining the value of the slope, the value of β contained in σ can be determined since all other components of the term R^c_S σ/ϕ are known for a particular filter run.

Figure 25. Theoretical plot of head loss versus X for flat septa

 $\sim 10^{-10}$

With cylindrical septa the initial dilution rate and also the in-place bulk density, $\gamma_{\sf p}$, of the filter aid must be known before the true value of β can be calculated.

The effects of using the wrong value of δ when calculating β from the results of a filter run with cylindrical septa are shown in Figure 26. If too high a value of 6 is used, the effects of initial dilution are not sufficiently accounted for as shown by the initial part of the curve in Figure 26. Also, the decrease caused by initial dilution in the amount of filter cake formed and increase in surface area is not accounted for which results in an increased slope of the later part of the curve in Figure 26. Using too low a value of 6 causes opposite effects.

The value of β is calculated from the slope of the linear regression line for a plot of H versus $\ln(1+R_g\phi X/R^2)$. If the data collected during the early part of a filter run are neglected or the data from a very long filter run are used, the calculated value of β will always be higher than the true β when too high a value of δ is used and lower than the true value of β when too low a value of δ is used. However, including the data from the early part of the filter run decreases the slope of the regression line and the value of the calculated β for the case where δ is too high and increases them when δ is too low. Thus, the calculated value of β may be higher or lower than the true value depending on

Figure 26. Theoretical plot of head loss versus natural log term for cylindrical septa showing the effects of using wrong values of ô

Figure 27. Theoretical plot of head loss versus natural log term for cylindrical septa showing the effects of using wrong values of γ_{r}

such factors as the length of the run, the actual dilution rate, the actual β , the body feed rate, and the size of the septa. This is demonstrated in Table 7. Values of head loss for elapsed times of filtration of 5, 30, 60, 90, ..., 300 minutes were computed for both a 1.0 inch and a 3.5 inch

Table 7. Effect of using wrong δ on values of β found by regression analysis

$\delta / h r$	$\beta, 10^6$ ft ⁻²	s_E , ft H_2O	R , $\frac{8}{3}$	% error
1.0 inch diameter septum				
$\overline{\mathbf{c}}$ $\bf 4$ ϵ 8 10 12 14 16 18 ₁ $_{\infty}a$	10.289 10.056 10.017 10.006 10.000 10.003 10.003 10.005 10.006 10.041	0.435 0.169 0.074 0.027 0.000 0.017 0.029 0.036 0.042 0.057	99.807 99.971 99.994 99.999 100.000 100.000 99.999 99.999 99.998 99.997	$+2.89$ $+0.56$ $+0.17$ $+0.06$ 0.00 $+0.03$ $+0.03$ $+0.05$ $+0.06$ $+0.41$
3.5 inch diameter septum				
2468 10 12 14 16 $18\atop \infty a$	10.427 10.111 10.042 10.016 10.000 9.996 9.992 9.989 9.988 9.995	0.558 0.215 0.094 0.034 0.000 0.021 0.036 0.045 0.052 0.073	99.838 99.976 99.995 99.999 100.000 100.000 99.999 99.999 99.999 99.997	$+4.27$ $+1.11$ $+0.42$ $+0.16$ 0.00 -0.04 -0.08 -0.11 -0.12 -0.05

 a_t used in place of X.

diameter septum with the following hypothetical conditions:

 $\delta = 10/hr$

 $\gamma_{\rm p}$ = 16 lb/cu ft $3 = 10 \times 10^6$ ft⁻² $C_p = 100$ mg/1 $q = 1.00$ gpm/sq ft $w = 0.10$ lb/sq ft

and

temperature = 60 °F

Values of β using different values for δ were then calculated from the slopes of linear regression lines (Figure 28) of H versus $\ln(1 + R^{\alpha}_{\alpha}X/R^2)$. The results of these calculations show that using the wrong value of δ , or even neglecting initial dilution by using time in place of X, does not cause serious errors in calculating B. For very short filter runs, especially if data are not collected beyond the period of initial dilution, the errors are greater. In such cases, a good estimate of δ is needed. For a completely mixed system, the initial dilution rate is theoretically equal to the flow rate divided by the volume of the filter housing. Since it has been found that the theoretical dilution rate does not always account for observed effects attributed to initial dilution, it is recommended that a method for estimating ô to fit observed data be used (27). In this method, the assumption is made that the inflection point of the head loss versus time curve occurs when $\delta t = 3$. When $\delta t = 3$, the term $(1 - e^{-\delta t}) = 0.95$ and the

 ~ 10

 $\label{eq:3} \gamma = 1 - \frac{1}{2} \sum_{i=1}^{2^k-1} \frac{1}{2^k} \, .$ $\label{eq:1} \left[\begin{array}{cc} \ddots & \ddots \\ \ddots & \ddots \\ \end{array}\right] \begin{array}{ll} \ddots & \ddots \\ \ddots & \ddots & \ddots \end{array}$

 $\sim 10^6$

 $\mathcal{L}_{\mathcal{A}}$

 \sim \sim

concentrations of body feed and suspended solids in the filter housing should be 95 percent of the concentrations in the influent if the contents of the housing are completely mixed (See Equation 54, Appendix B). It is assumed that at this point, the increase in the rate of head loss development caused by the increasing concentrations of body feed and suspended solids in the filter housing is balanced by the decrease in the rate of head loss development caused by the increasing surface area of the filter septums. By estimating the time of inflection, t^1 , from a plot of head loss versus time, 6 may be estimated as:

$$
\delta \approx 3/t_i \tag{32}
$$

The effects of using the wrong value of γ_p when calculating 6 from the results of a filter run with a cylindrical septum filter are shown in Figure 27. If the value of $\delta_{\mathbf{p}}$ used is too high, the actual increase in septum area that occurred during the filter run is not completely accounted for. Therefore, at any particular time, the observed head loss is lower than the theoretical head loss required for the calculated β to be equal to the actual β . Thus, the calculated value of β will be lower than the actual value of If the value of Y_p used is too low, the calculated value of 6 will be higher than the actual value. This is demonstrated by the results presented in Table 8. The same

hypothetical data used in preparing Table 7 were used to prepare Table 8. The results show that using the wrong value of $\gamma_{\mathbf{p}}$ may cause serious error in the value of β calculated, especially when small diameter septa are used. Therefore, an accurate value of γ_p should be obtained before β is calculated. A standard procedure for measuring γ_{p} has been proposed by Baumann and Oulman (6). Values of γ_p for several grades of perlite and diatomite filter aids have been given in Table 3 (p. 29).

BID program

A user manual for a computer program for Beta Index Determination or the BID program is given in Appendix D. The program allows calculation of the β index from the results of filtration with flat or cylindrical septa.

The main use of the BID program has been to analyze the results from hundreds of filter runs made during research studies. If only a few filter runs are made, it is just as easy to calculate values of β manually; especially if the filter runs are made using flat septa such as used in the SSCR filter. Even with cylindrical septa it is not difficult to calculate β indices manually for several filter runs if many of the variables $(q, R_c, w, etc.)$ are the same and head loss is measured at the same times for every filter run. In fact, it is recommended that manual calculations be made so that any errors in the analysis will be immediately noticed.

Table 8. Effect of using wrong $\gamma_{\mathcal{D}}$ on values of β found by regression analysis ^{'p}

 $\hat{\boldsymbol{\gamma}}$

 $\overline{}$

Examples of the manual calculation of β from results using both flat and cylindrical septa are given in the BID Program User Manual in Appendix D.

S Index Prediction

Empirical prediction equations

 β prediction equations have been presented most often in the form of Equation 15:

$$
\beta = 10 \t{b} \t{1} (C_{S}/C_{F}) \t{2}
$$
 (15)

As discussed previously, this equation was developed by reasoning that filter cakes containing equal ratios of suspended solids to body feed should have equal resistances per unit weight of cake (5). It was then discovered that when β was plotted versus $C_{\rm g}/C_{\rm F}$ on log log graph paper, a straight line was formed which corresponds to Equation 15. The values of b_1 and b_2 can be determined from such a plot since the slope is equal to b₂ and when $\texttt{C}_\texttt{S}/\texttt{C}_\texttt{F}$ is 1.0, 10 $^{\texttt{D}}$ equals β . Also, Equation 15 can be transformed to a linear equation by a logarithmic transformation; i.e., log $\beta = b^2 + b^2$ b₂ log(C_S/C_F). Therefore, b₁ and b₂ can be determined by linear regression of log β and log(C_g/C_p).

The Equation 15 form of the β prediction equation has been found to be acceptable by several investigators because either C_S was held constant or because the suspended solid did not show a sufficient concentration effect. If C_S is not varied, Equation 15 can be used to predict β for waters containing different amounts of the suspended solid than the water used for collecting data only if it is assumed that

concentration effects are negligible (i.e., the same C_S/C_F always results in the same β). If this assumption is not valid, Equation 15 can only be used when C_S is equal to that used for collecting the data. In such a case Equation 27 is equally valid:

$$
\beta = 10^{b} \, \text{C}_{\text{F}}^{b} \tag{27}
$$

For example, data presented by Baumann et al. (8) for the filtration of water containing $8.0 + 0.5$ mg/l of iron were used to develop 3 prediction equations of the form of Equation 27. Equations were developed for each of the 11 filter aids used and the values of $b₂$ were found to vary from 1.81 to 2.15. In fact, if $b₂$ is set equal to 2.00, a plot of β versus $1/C_{\rm p}^2$ yields a straight line which passes through the origin and has a slope equal to 10^{-1} . Typical results are shown in Figures 29 and 30.

Waters which have been shown to exhibit negligible concentration effects are: University tap water containing unsettled Ball clay (61), University tap water containing settled Ball clay (3), distilled water containing unsettled Ball caly (3), distilled water containing settled Ball clay (3), and several coagulated and settled surface waters (15). For such waters, the prediction equation represented by Equation 15 can be used.

The only waters definitely shown to exhibit pronounced

Figure 29. Typical result of plotting β versus $1/C_F^{-2}$ with data from Baumann et al. (8) C_S const at 8.0 mg/1 iron

 $\label{eq:1} \frac{1}{2}\sum_{\mathbf{k}\in\mathbb{Z}^d}\left\langle\frac{\partial}{\partial\mathbf{k}}\right\rangle\left\langle\frac{\partial}{\partial\mathbf{k}}\right\rangle\left\langle\frac{\partial}{\partial\mathbf{k}}\right\rangle\left\langle\frac{\partial}{\partial\mathbf{k}}\right\rangle\left\langle\frac{\partial}{\partial\mathbf{k}}\right\rangle\left\langle\frac{\partial}{\partial\mathbf{k}}\right\rangle\left\langle\frac{\partial}{\partial\mathbf{k}}\right\rangle\left\langle\frac{\partial}{\partial\mathbf{k}}\right\rangle\left\langle\frac{\partial}{\partial\mathbf{k}}\right\rangle\left\langle\frac{\partial}{\$

 $\ddot{}$

Figure 30. Typical result of plotting β versus with data from Baumann et al. (8) C_S constant at 8.0 mg/l iron

 \mathbb{R}^2

concentration effects are University tap water containing Wyoming bentonite clay (3, 61) and University tap water containing iron floe (3). There are indications that effluent from the lime-soda ash softening process also exhibit concentration effects (15, 27), however the data available for these effluents are field data collected without substantial variations in C_S .

The most complete data for developing a β prediction equation for University tap water containing Wyoming bentonite clay were collected by Regunathan (61) . For a water such as this, which exhibits a definite concentration effect, it is recommended that a B prediction equation of the following form be used:

$$
\beta = 10^{b_1} \, c_S^{b_2} \, c_F^{b_3} \tag{33}
$$

Regunathan's data for tap water plus Wyoming bentonite clay (Series D runs) are contained in Appendix A (Table 25). If a log log plot of β versus C_F is made (Figure 31), the value of b^2 can be determined as the slope of the straight line drawn through points for which C_S values are the same. The value of b^2 for Regunathan's data was found to be -2.46 which is the slope of the straight line drawn through the 5 points for which $C_{\rm g}$ was approximately 85 JTU. If it is then assumed that the value of b^2 is the same for all values of C_S , parallel lines can be drawn through points collected at
Figure 31. Log β versus log C_F for University tap water containing Wyoming bentonite clay. Data from Regunathan (61)

the other values of C_S (dashed lines in Figure 31). From these lines, values of β at the same value of C_p can be determined for each value of C_S at which data were gathered. In the example, the following values of β were determined at $C_F = 600 mg/l$:

These values were then used to make the log log plot of β versus C_S shown in Figure 32. The slope of the straight line in Figure 32 is b_2 which was found to have a value of $+1.31$. Now, $\beta = 29.9 \times 10^6 \text{ ft}^{-2}$ when $C_S = 85$ JTU and $C_F = 600 \text{ mg/l}$. Therefore :

$$
29.9 \times 10^{6} = 10^{b} \cdot (85)^{1.31} (600)^{-2.46}
$$

and taking the log of both sides of the equation:

 $7.476 = b_1 + (1.31 \times 1.929) + (-2.46 \times 2.778)$

$$
b_1 = 7.474 - 2.527 + 6.834
$$

$$
= 11.783
$$

and

Therefore, for University tap water containing Wyoming bentonie clay:

Figure 32. Log β versus log C_S for University tap water containing Wyoming bentonite clay. Data from Regunathan (61)

 \mathcal{L}

$$
\beta = 10^{11.783} \text{ C}_\text{S}^{-1.31} \text{ C}_\text{F}^{-2.46}
$$

Equation 33 can also be transformed to a linear equation by a logarithmic transformation; i.e., log $\beta = b^1 + p^2$ log(C_S) + b^2 log(C_F), and b^1 , b^2 , and b^3 can thus be determined by linear regression of log β , log(C_S), and log(C_F). This was done using Regunathan's data with the following result:

$$
\beta = 10^{11.6685} \text{ C}_\text{S}^{1.2940} \text{ C}_\text{F}^{-2.4054}
$$

which agrees with the prediction equation calculated manually. Similar calculations are made in Appendix E for the results from filtering iron floe.

If Equation 33 is used for predicting β for waters which do not exhibit concentration effects, then $b₂$ should equal $-b₃$ so that the equation can be written as Equation 15.

Several theories have been proposed to explain why concentration effects occur. Some of the theories presented in chemical engineering literature pertaining to constant pressure filtration of concentrated slurries have recently been discussed by Arora (3). The only one of these theories which might possibly explain the decrease in β with increases in C_S and C_F observed for constant rate filtration of water is the theory that at higher concentrations, there is interference or crowding between particles as they are laid on the surface of the filter cake (38). Because of this

interference, the particles do not form as dense a cake as they would if they had more freedom of movement. This theory gives a qualitative explanation of the concentration effects that have been observed; however, it does not explain why concentration effects have been observed with some suspended solids but not with others, even though the same types and approximate amounts of filter aids were used.

Regunathan (61) thought that the concentration effects observed with tap water containing Wyoming bentonite clay were due to swelling of the montmorillonite clay mineral within the filter cake. Regunathan agitated the clay slurry for 15 to 21 hours before a filter run was made so it is doubtful if the clay would swell within the cake. This theory also does not explain the concentration effects observed with iron-bearing waters or lime-soda ash process effluents.

Dillingham (27) proposed that concentration effects may be due to the use of the β index rather than specific resistance based on the weight of filter aid or due to errors in the assumption that the suspended solids do not increase the cake thickness. Neither of these explanations indicate why concentration effects are not observed for all suspended solids or why they are observed with flat septa as well as cylindrical septa. Also, there are several articles in the literature which report concentration

effects when specific resistance is calculated (38, 65, 69). Dillingham also thought that the concentration effects that Regunathan observed could have been the result of using turbidity in place of suspended solids concentration for C_c . It has since been shown that the effects are still observed when suspended solids concentrations are used.

No one has yet shown proof as to whether concentration effects are due to the concentration of solids (suspended solids plus body feed) per se or if they are due to changes in the physical properties of the suspended solid particles caused by changes in concentration. This may explain why some solids exhibit concentration effects while others do not.

Rational prediction equation

A relation between the β index and the physical properties of the filter cake can be derived by equating the precoat filtration equations to the Kozeny-Carman equation for the head loss for laminar flow through a uniform bed of solids. The Kozeny-Carman equation was derived from Darcy's law by assuming that a granular bed is equivalent to a group of identical, parallel channels such that the total internal surface area and volume are equal to the particle surface area and volume of voids, respectively (18). This derivation is presented in most textbooks on unit operations (46, 62).

The equation may be written as:

$$
\frac{H_C}{L_C} = 36k \frac{qv}{g} \frac{(1-\epsilon)^2}{\epsilon^3} (\frac{1}{\psi \bar{d}})^2
$$
 (34)

where :

- d = mean spherical diameter of particles
- ψ = sphericity, defined as the ratio of the surface area of the equivalent-volume sphere to the actual or true surface area (33). For spherical particles, $\psi = 1.0$, and for all other shapes, ψ is less than unity.

 $k = constant$

Carman (19) found that the value of k was about 5.0 so that 36k = 180. More recently, Ergun (32) reviewed data in the literature and found that $36k = 150$.

Now consider Equation 8 for the head loss through a flat filter cake:

$$
H_{\alpha} = \sigma X \tag{8}
$$

Since :

$$
\sigma = q^2 v \beta C_F / g \text{ by definition}
$$

$$
\beta = a_C \frac{\gamma_w}{\gamma_p} (10^{-6}) \text{ by definition}
$$

and :

$$
q = \frac{Q}{A}
$$

then Equation 8 can be written as:

$$
H_C = a_C \frac{qv}{g} \left[\frac{Q}{A} C_F \frac{\gamma_w}{\gamma_p} (10^{-6}) x\right]
$$

Note that the term in brackets is equivalent to the thickness, L_{c} , of the filter cake since:

 $QX = cu$ ft of water filtered

 $QX\gamma_{\rm M}$ = lbs of water filtered

and :

$$
C_F =
$$
 lb filter aid/10⁶ lb water
 $C_F(10^{-6}) =$ lb filter aid/lb water

so that:

$$
\mathrm{QXY}_{\mathbf{W}}\mathrm{C}_{\mathbf{F}}\left(10^{-6}\right) \;=\; \mathrm{lb}\;\,\mathrm{filter}\;\,\mathrm{aid}\;\,\mathrm{deposited}
$$

and :

$$
\frac{QXY_WC_F(10^{-6})}{AY_p} = \frac{\text{cu ft deposited}}{\text{unit area}} = L_c
$$

Therefore :

$$
\frac{H_c}{L_c} = a_c \frac{qv}{g}
$$
 (35)

The right hand side of this equation may now be equated to the right hand side of the Kozeny-Carman equation (Equation 34) which leads to:

$$
a_{\rm c} = 150 \frac{(1-\epsilon)^2}{\epsilon^3} \left(\frac{1}{\psi d}\right)^2 \tag{36}
$$

and from the definition of the 6 index:

$$
\beta = 150 \quad (10^{-6}) \quad \frac{\gamma_{\text{w}}}{\gamma_{\text{p}}} \quad \frac{(1-\epsilon)^2}{\epsilon^3} \quad (\frac{1}{\psi \mathbf{d}})^2 \tag{37}
$$

For the filtration of a dilute suspension of a nonhomogeneous mixture of suspended solids and filter aid. Equation 37 appears to have little practical merit. Arora (3) presented Table 9 which shows the large variation of the factor $\epsilon^3/(1-\epsilon)^2$ with only slight variations in ϵ . The difficulties in determining ε , ψ , and d of a dirty filter cake make this method of predicting 6 too impractical.

Table 9. Variation of permeability factor, $\varepsilon^3/(1-\varepsilon)^2$ with porosity e (3)

ϵ	$\epsilon^3/(1-\epsilon)^2$	
0.90 0.85 0.80 0.78 0.75 0.70 0.65 0.57 0.50	72.9 27.3 12.8 10.0 6.7 3.8 2.2 1.0 0.5	

Equation 37 does, however, have some theoretical merit. It has been theorized that an empirical prediction equation such as:

$$
\beta = 10^{b} \text{ } (C_{S}/V_{V})^{b} \tag{19}
$$

would have the same exponents for all filter aids when the same suspended solid was filtered (54). Consideration of Equation 37 reveals that even if the permeability factor is

accounted for, there are still considerable differences in the bulk densities, sphericities, and mean particle sizes of the filter cakes formed using different filter aids.

MAIDS program

A user manual for a computer program for determination of 6 prediction equations by MAnipulation and Interpretation of Data Systems or the MAIDS program is given in Appendix E. The program was written as a general program to perform linear regressions and to transform and print out data. The main use of the MAIDS program has been for determining β prediction equations. The coefficients of any prediction equation that is, or can be transformed to, a linear equation with from 2 to 8 variables, can be determined using MAIDS. Any desired transformation, such as a logarithmic transformation, is possible with MAIDS.

Examples of the determination of β prediction equations are presented in the user manual. Examples of estimation of the regression coefficients by graphical methods are also given.

CALCULATION OF OPTIMUM DESIGN CONDITIONS

General

The general procedure for computer calculation of the optimum design conditions is to calculate the total filtration cost for each combination of q, C_F , and H_t with q, C_F , and H_t varied by desired increments over specified ranges and then picking out that combination which results in the least total cost. For example, if q is varied by 0.1 gpm/sq ft from 0.2 to 2.5 gpm/sq ft, C_F is varied by 10 mg/l from 20 to 100 mg/l, and H_t is varied by 10 ft from 50 to 150 ft there are 21 x 9 x 11 or 2079 combinations for which the total cost is calculated. That combination which results in the least total cost is the optimum.

With a high-speed, digital computer these calculations can be made in a matter of seconds. For manual calculation, the total number of combinations for which the total cost is calculated can be greatly reduced by using the graphical techniques presented by LaFrenz (see p. 68). If manual calculations are made, the ranges of values of q, C_p , and H_t and therefore the number of calculations required, can be reduced if a good approximation of the optimum q, C_p , and H_f combination is made beforehand. To aid in making this approximation, the effects of several of the filtration variables and cost factors on the optimum design conditions

are discussed later in this chapter.

Initial Dilution

The filter run length must be calculated so that the number of filter runs that can be made in a certain length of time can be determined. The number of filter runs must be known in order to calculate the cost of precoat filter aid and the increase in filter area required to provide filtered water for backwashing.

The filter run length can be calculated using Equation 8 for flat septa or Equation 6 for cylindrical septa. Dillingham (25, 27) neglected initial dilution when calculating t by approximating X by t since when X is used there are not explicit solutions for t of either Equation 8 or Equation 6. Arora (3) has stated that this can result in serious error. Initial dilution can be considered and explicit solutions of Equations 8 and 6 obtained by replacing X with $t - t^2$ since $X = t - t_d$ when t is large (Equation 30). The errors in doing this to calculate the filter run length are negligible since X approaches $t - t_d$ very rapidly. For example, if δ is only 1.0/hr or $t^{}_{d}$ = 1.0 hr, X is equal to t - 0.950 hr when t is only 3 hr. A more practical example occurs when $\delta = 10/hr$ or $t^1_A = 0.1$ hr. In this case, X = t - 0.099 hr when t is only 0.5 hr and $X = t - 0.099995$ hr when t is only 1.0 hr. In most cases, the length of filter runs for optimum design will be

greater than 6 hr.

Several filter runs were made by the author and Madan L. Arora using the SSCR filter. The primary purpose of this study was to evaluate the ability of the SSCR filter to predict the results of large-scale filters. These results have been reported by Arora (3) and are presented in Appendix A (Table 32). Several different suspensions were filtered. With each suspension, filter runs were made with the same value of $C_{\rm g}/C_{\rm F}$ but various values of $C_{\rm g}$ and $C_{\rm F}$. This was done to study the effects of the solids concentration on the 6 index and a discussion of the results was made by Arora (3).

During the course of these studies it became apparent that the suspended solids concentration, and/or the body feed concentration since C_g/C_p was held constant, had a marked effect on the observed initial dilution rate and apparent detention time. The head loss versus time curves for one series of filter runs (Runs 55-60) with unsettled ball clay in distilled water and Hyflo Super-Cel filter aid are shown in Figure 33. Notice that the apparent detention time varies from about 3 min to 46 min. Theoretically, the detention time should be a constant of about 1.2 min based on the filter volume and filtration rate. Furthermore, it was found that a log log plot of C_S , (or C_F since C_S/C_F was held constant) versus the apparent detention time was a

straight line (Figure 34) corresponding to the equation:

$$
t_a = b_1 C_S^{b_2}
$$
 (38)

where :

- b_1 and b_2 are constants.
- t_a = apparent detention time observed during a filter run.

It is theorized that the apparent detention time varies with the solids concentration because at the beginning of a filter run a definite amount of time is required for a filter cake to form. Before the cake is formed, the majority of the suspended solids removal is at the surface of the precoat layer. As the surface of the precoat layer becomes "plugged", a greater percentage of the suspended solids are removed resulting in the observed increase in the rate of head loss increase. After a filter cake is formed, the rate of head loss increase is constant for flat septa.

According to this theory, the quality of the filter effluent should continually improve during the early part of a filter run until the filter cake is formed. This was observed in this study. For example, during Filter Run 59 which is included in Figure 33, the effluent turbidity was 2.7 JTU after 14 min of filtration, 0.06 JTU after 32 min, and 0.03 JTU after 44 min. From Figure 33 it appears that

Figure 33. Plots of head loss versus time of filtration for filter runs with unsettled Ball Clay in distilled water using Hyflo Super-Cel as a filter aid at $C_g/C_F = 0.495$

 \sim

Filter runs 55-60 in Appendix A, Table 32

 \sim

Figure 34. Log C_S versus log t_a for data from the series of filter runs shown in Figure 33 Suspension = unsettled Ball clay in distilled water Filter aid: Hyflo Super-Cel

 \sim

 $\ddot{}$

 $\ddot{}$

the cake was completely formed at about 30 min. Influent turbidity during this filter run was about 45-48 JTU.

The proposed theory would lead one to predict that if the same concentrations of suspended solids are filtered, a suspended solid such as clay $(1-2 \mu \text{ diameter})$ which is less readily removed by straining at the surface of the precoat layer will result in a longer apparent detention time than a suspended solid such as iron floc $(20-30 \mu$ diameter) which is more readily removed by straining. Similarly, for the same suspended solid, the use of a coarse filter aid should produce longer apparent detention times than a fine grained filter aid. These predictions are borne out by the results shown in Figure 35. The apparent detention times when filtering settled Ball clay were approximately 15 times the apparent detention times observed when filtering iron floe. Filter runs were not made using different filter aids with the same suspended solid; therefore, there is no direct evidence to show the effects of different filter aids. However, a few filter runs were made using a Millipore filter (0.45μ) pore size) in place of a precoat layer. The apparent detention time for these filter runs were 5-10 times less than those observed when the same suspension was filtered using a precoat of Hyflo Super-Cel.

If it is assumed that the same amount of a particular suspended solid is always required to plug precoats of the

Figure 35. Log C_c versus log t_a for data from the filtration of settled Ball clay in distilled water and iron floe in University tap water

 $\hat{\mathcal{L}}$

1000 SYMBOL C_S/C_F 500 \circ 0.380 \Box 0.533 0.233 Δ 0.240 $\overline{\nabla}$ 0.295 \Diamond 0.384 $+$ IRON FLOC 100 BALL CLAY 50 PRECOAT $+$ O) E *w>* U **** MEMBRANE 10 $\ddot{\diamond}$ **** 5 ◇ $\frac{1}{1}$ $\overline{5}$ 10 50 TOO $\mathbf{r}_{\mathbf{q}}$, min

same filter aid, then one can write:

$$
c_{S} t_{a} q - c_{p} = a_{1}
$$

where :

 C_p = amount of suspended solid that passes through the filter

 a_1 = constant

If it is further assumed that the same amount of suspended solid always passes through the filter (i.e. C_p is independent of C_S) then:

$$
C_S \tfrac{1}{2} q = a_2 \tbinom{39}{}
$$

where :

 a_2 = constant.

The filter runs made by the author and Arora were all made using q equal to 1.05 gpm/sq ft. For the results of this study, the following equation should apply:

$$
t_a = b_1 c_s^{-1}
$$

which is identical to Equation 38 with $b₂ = -1$. The values of b_1 and b_2 for the results of this study are given in Table 10. Obviously b^2 is not equal to -1 as theorized. The assumptions that were made are very broad, however the theory is presented here to indicate that $t^{}_{\scriptscriptstyle{A}}$ is inversely related to c_S and q and since the data were collected with q

held constant, one must not assume that t_a does not also vary with q. Arora (3) expressed the opinion that t^2 may also be a function of C_F . It is the opinion of this author that $t^{}_{\text{a}}$ varies very little with $C^{}_{\text{F}}$. The data shown in Figure 35 for the filtration of a suspension of settled Ball clay in distilled water using Hyflo Super-Cel filter aid were collected at five different values of C_S/C_F . No apparent effect of C_F on t_a was observed. Filter run 12 was made with C_S/C_F = 21.3/40 = 0.53 and Filter run 14 was made with C_S/C_F = 21.0/90 = 0.233. The apparent detention times observed for

these two filter runs were 20.5 min and 19.0 min. respectively. The two points which fall the furthest from the straight line are both for filter runs with $C_g/C_p = 0.295$.

For the filter runs shown in Figure 33, $t_a = 138 C_S^{-0.73}$. This equation was used to predict $t^{}_{\overline{a}}$ for each of these filter runs. The predicted values of $t^{}_{\text{a}}$ and the observed values of β were used to calculate the head loss versus time curves for each of these filter runs. The results are shown in Figure 36. The empirical equation for predicting t^2 gives excellent results.

Total Cost Calculations

The necessary steps for calculating the total cost of filtration for a particular combination of q , $C_{\mathbf{F}}$, and H_{+} are outlined below. These steps are the same for both computer and manual calculation although for manual calculation some simplifying assumptions can be made. The costs are expressed in \$/month and units of pounds, feet, and hours are used. Sample calculations are included in Appendix F.

3 index

The β index is calculated using Equation 33:

$$
\beta = 10^{b_1} \, c_S^{b_2} \, c_F^{b_3} \tag{33}
$$

This equation is equivalent to Equation 15 by letting

Figure 36. Plots showing predicted head loss versus time curves and observed data for filtration runs with unsettled Ball Clay in distilled water using Hyflo Super-Cel as a filter aid at $C_{\rm g}/C_{\rm F}$ = 0.495

Filter runs 55-60 in Appendix A, Table 32

Apparent detention time was predicted by the equation:

 $t_a = 138 \text{ C}_s$

 \sim

 $b_2 = -b_3$ and to Equation 27 if b_2 is set equal to 0.

Filter run length - detention time considered

The head loss through the precoat layer is calculated from Equation 5:

$$
H_p = q\nu \xi w / g \tag{5}
$$

and the head loss through the filter cake is calculated as:

$$
H_C = H_L - H_p \tag{40}
$$

The head loss through the precoat is very small compared to the head loss through the filter cake. Therefore, for manual calculations, H_c can be considered equal to H_c .

By replacing X by $t - t_a$ in Equations 8 and 6, the length of the filtering cycle, t_f, can now be calculated as:

$$
t_f = \frac{H_c}{\sigma} + t_a \tag{41}
$$

for flat septa, and:

$$
t_{f} = \frac{(e^{H_{c}\phi/R_{s}\sigma} - 1) R_{o}^{2}}{R_{s}\phi} + t_{a}
$$
 (42)

for cylindrical septa. The total length of the filter run is then equal to the length of the filtering cycle plus the estimated time required to backwash and precoat the filter.

Filter run length - detention time ignored

In order to include detention time in the filter run length, it is necessary to know the detention time for every value of filtration rate that is considered. The apparent detention time is thought to be due to 1) the detention time attributed to initial dilution in the filter housing, and 2) the detention time attributed to the time required for precoat plugging and cake formation. The detention time due to initial dilution is theoretically equal to V_f/Q . The flow rate, Q, is constant. However, the volume of the filter housing, V_f , will probably vary with the area of the filter septa and therefore, the filtration rate, q. According to the theory presented for developing Equation 39, the detention time due to precoat plugging and cake formation is inversely related to the filtration rate. No data are available to substantiate this.

It is doubtful that the apparent detention time is simply equal to the detention time due to initial dilution plus that due to precoat plugging and cake formation. The detention time due to precoat plugging and cake formation is probably a function of the initial dilution rate. Obviously, more research is needed before initial dilution and detention time will be fully understood.

Under optimum design condititions, filter run lengths are quite long. If the concentration of suspended solids is

reduced the apparent detention time will be longer, but the optimum run length will also be increased. Also, the filter run length is only used to determine the number of filter runs per month which is needed for calculating the cost of precoat filter aid and the increase in filter area required to produce water used for backwashing. The cost of precoat filter aid and the cost ascribed to backwashing are usually a minor part of the total cost. They are only significant for very short filter runs which do not provide optimum conditions. Therefore, it is the opinion of this author that detention time can be ignored without seriously affecting the calculation of optimum design conditions. For example, the data collected by the author for a suspension of settled Ball clay in distilled water were used to develop the B prediction equation:

$$
\beta = 10^{8.31} \, \text{C}_{\text{S}}^{2.12} \, \text{C}_{\text{F}}^{-2.17}
$$

This equation was then used in the determination of the optimum design conditions for a 1 MGD plant and suspended solids concentrations of 3 mg/1 (t^2 =72.0 min), 20 mg/1 $(t^2_a=20.5 \text{ min})$, and 100 mg/l $(t^2_a=7.2 \text{ min})$. The apparent detention times were taken from Figure 35 and identical calculations were also made with apparent detention times ignored $(t^2)^{=0}$. It was assumed that t^2 varied inversely with q (Equation 39). The cost data presented by Dillingham (27)

were used for these analyses and the results are presented in Table 11.

C_S mg/1	t_a min	σ	Optimum conditions H_{t}	$\overline{c_{_{\rm F}}}$	t_f hr	Total cost $\frac{2}{2}$ /MG
3	72.0	1.60	30	14	17.9	34.2
	0.0	1.60	30	14	17.1	34.5
20	20.5	0.80	140	30	14.7	65.3
	0.0	0.80	140	30	14.3	65.7
100	7.2	0.40	150	110	9.7	145.8
	0.0	0.40	150	110	9.4	147.3

Table 11. Effect of apparent detention time on optimum design conditions

Neglecting the apparent detention time did not have any effect on the optimum design conditions that could be observed with the incremental values of q , H_t and C_F used (0.2 gpm/sq ft, 5 ft and 5 **mg/A,** respectively). Therefore, the length of the filtering cycle can be calculated by assuming $t^{\ }_{a}$ = 0 and using Equation 41 for flat septa and Equation 42 for cylindrical septa. The total run length is again equal to the length of the filtering cycle plus the estimated time for backwashing and precoating.

Filter area

The filter area required can be calculated as:

$$
Area = \frac{QGPM'}{q}
$$
 (43)

where :

QGPM' = flow rate in gpm required to meet both demand and backwashing requirements

$$
= \frac{\text{QMGD}^{\prime} \times 10^6}{1440 - n(BWT)}
$$

where :

n = number of filter runs per day

$$
= \frac{24 \text{ hr/day}}{\text{filter run length}}
$$

BWT = time required per filter run for backwashing and precoating, hr

and :

QMGD' = flow rate in MGD required to meet both demand and backwashing requirements

$$
= QMGD + (\frac{n \text{ (Area)} (\text{BWGSF})}{10^6})
$$

where:

- QMGD = design flow rate in MGD required to meet demand requirements
- $BWGSF = amount of water required to backward the filter$ in gal/sq ft of filter area

Since the filter area is dependent on QGPM' and QGPM' is dependent on the filter area, an iterative calculation

process is called for. This can be done as follows:

- 1. Assume QMGD' = QMGD
- 2. Calculate QGPM'
- 3. Calculate Area
- 4. Calculate QMGD' and repeat steps 2 and 3

The above process is continued until the areas calculated in successive iterations do not differ by more than one percent. In most cases only two or three iterations will be required- More iterations are required when the filter run length is very short.

First cost

The total first cost can be calculated by multiplying the area by the first cost in \$/sq ft obtained from the plot of first cost versus filter area. The total cost should be multiplied by the rate factor if the filtration rate is different than that for which the first cost data were obtained. The first cost is amortized over the design life of the plant by the equation:

CF per year =
$$
\text{Tr}(\frac{i[(1+i)^{n} - SV/100]}{(1+i)^{n}-1}
$$
 (44)

and

CF per month =
$$
\frac{CF}{12}
$$

where :

 $CF =$ amortized first cost, \$

TFC = total first cost, $$$ i = interest rate n = design life, yr SV = salvage value, % first cost

Labor and maintenance cost

Both labor and maintenance costs are assumed to vary primarily with the filter area and are therefore combined. This cost can be calculated by multiplying the filter area by the cost of labor and maintenance in \$/sq ft per month obtained from the plot of labor and maintenance cost versus filter area. The cost should be multiplied by the rate factor if the filtration rate is different than that for which the labor and maintenance cost data were obtained.

Filter aid cost

The amount of precoat filter aid used in lb/month is equal to:

$$
PFA = w(Area)N \qquad (45)
$$

where N is the number of filter runs per month and is equal to 24 hr/day x 30.4 days/month divided by the filter run length.

The amount of body feed filter aid used in lb/month is equal to:

$$
BFA = C_F \quad (QMGMO') \quad 8.33 \tag{46}
$$

where QMGMO' is the flow rate in MG per month required to meet demand and backwashing requirements and is equal to QMGD' X 30.4 days/month.

The total cost of filter aid per month is then equal to:

$$
CFA = \frac{PFA + BFA}{2000} \times \frac{\zeta}{\tan} \tag{47}
$$

Power cost

The amount of power used per month can be calculated using Equation 29. If QMGMO' is used, the equation becomes:

$$
P = \frac{QMGMO^{\bullet} \times H_{\text{t}}}{E} \quad (8.33)
$$
 (48)

The power cost per month then equals P multiplied by the unit cost in \$/kwh.

Total and operating cost

The operating cost is calculated as:

$$
COPER = CL + CM + CFA + CP
$$
 (49)

and the total cost is :

$$
CTOTL = CF + COPER
$$
 (50)

POPO Program

A user manual for a computer program called Program for Optimization of Plant Operation or the POPO program is given in Appendix F. This program reads in all of the
necessary filtration data and cost information and computes the total filtration cost for all desired combinations of q, H_+ , and C_F . The 10 combinations which result in the lowest total costs are printed out along with other filtration and cost information. These results are computed and printed out for β indices equal to 50, 75, 100, 125, 150, and 175 percent of those predicted by the S prediction equation. Results for different percentages of the β index are included because the actual β may vary from the predicted value depending on the accuracy of the prediction equation and to indicate how the optimum design conditions may vary with changes that might occur in the characteristics of the filter influent.

An example of the manual calculation of the total filtration cost is also included in Appendix F.

Effects of Filtration and Cost Factors on Optimum Design Conditions

Several optimum design calculations were made using the POPO program to show how the optimum design conditions and total cost vary with certain filtration and cost factors. Those factors considered were:

- 1. Filter cake resistance
- 2. Filter aid cost
- 3. Suspended solids concentration

4. First cost

5. Power cost

6. Labor and maintenance cost

The design data shown in Table 12 were used for these analyses. The cost data used were obtained by Dillingham (27) from filter manufacturers and some existing filtration installations. These data were collected prior to 1965 and are presented here only for demonstration purposes.

Effect of filter cake resistance on optimum design conditions

From a review of the β prediction equations developed by Dillingham et al. (29) from data obtained by various investigators and those developed from recent data, it appears that the equation:

$$
\beta = 10^{b_1} (C_S/C_F)^2
$$

or

$$
\beta = 10^{b_1} c_S^{2} c_F^{-2}
$$

is appropriate for suspensions which do not exhibit concentration effects. Therefore, the relative cake resistance of these suspensions is indicated by the value of b_1 . The range of values of b_1 that have been observed is from approximately 7.0 to 11.0 (i.e., the cake resistance exhibited by the most resistant suspension is about 10^4 times that exhibited by the least resistant suspension for the same

Table 12. Basic data used to study the effects of various factors on optimum design conditions

 C_S/C_F ratio).

To show the effect of filter cake resistance, the optimum design conditions were computed for values of b_1 ranging from 7.0 to 11.0. The results are shown in Figure 37. It is observed that as the filter cake resistance increases, there is an exponential increase in the optimum body feed rate, terminal head loss, and total cost whereas the optimum filtration rate decreases. No limit was placed on the terminal head loss when making these calculations. In practice, the head loss is limited to about 150 ft. If head loss was limited to 150 ft in this example, the optimum design conditions for values of b_1 greater than 9.0 would be affected.

An estimate of the effect of an error in predicting β on the optimum design conditions can be obtained from Figure 3 7. For example, 15 filter runs were made using the SSCR filter (Appendix A, Table 32, filter runs 21-35) to determine the ability of the bench-scale filter to predict the results obtained with a large-scale pilot plant for the filtration of University tap water to which ferric chloride was added. The average difference between the values of β obtained with the SSCR filter and the pilot plant was 15 percent. This corresponds to a difference in the values of b_1 of only 0.06. From Figure 37, it is obvious that a 15 percent error in predicting the B index does not have any

Figure 37. Plots showing the effects of the relative cake resistance on optimum design conditions and total cost

 $\sim 10^6$

significant effect on the optimum design conditions.

Another observation that can be made from Figure 37 is the effect of the type and grade of filter aid on the optimum design conditions. Results presented by Baumann, et al. (8) for the filtration of University tap water plus ferric chloride $(C_S$ approximately 8.0 mg/1) with 11 different types and grades of filter aids produced by 4 manufacturers were fitted to the equation $\beta = 10^{12}$ $\left(C_{\rm g}/C_{\rm F}\right)^2$. The calculated values of b_1 are shown in Table 13. The values of b_1 for different grades of filter aid produced by any one manufacturer do not vary by more than 0.21. This small a change in b_1 does not cause very large changes in the optimum design conditions shown in Figure 37. Thus, the main factor which influences the filter cake resistance and the total cost is the filtering characteristics of the raw water. The total costs of filtering the various waters that have been studied at Iowa State University differ by as much as 500 percent whereas the largest difference in the total cost of filtering iron bearing water with different filter aids is about 30 percent. For a particular water, the type and grade of filter aid used is a significant factor. However, in future research, it may be more beneficial to study methods of reducing filter cake resistance by improving the filtering characteristics of the raw water rather than by improving the characteristics of filter aids.

	CLON OF ONLYCLOTCY COP WOLCH PIGS ICLERO CHEOLIGO		
	Filter aid designation	b_1	
S ₂		10.120	
S ₃		10.259	
S ₄		10.335	
J ₄		10.290	
J3		10.320	
J ₀		10.399	
E6		10.328	
E5		10.496	
E2		10.524	
G4		10.452	
Gl		10.585	

Table 13. Values of b, of $\beta = 10^{-1}$ (Cg/C_F)² for the filtration of University tap water plus ferric chloride

Effect of filter aid cost on optimum design conditions

The cost of filter aid is a significant portion of the total cost of precoat filtration and due to shipping costs, the unit price of filter aid may vary widely depending on the location of the proposed plant. To study the effect of filter aid cost, the optimum design conditions were determined for filter aid prices varying from \$60 to \$120 per ton. The data contained in Table 12 were used along with the β prediction equation:

 $\beta = 10^8$ C_S^2 C_F^{-2}

As shown in Figure 38, the optimum head loss and total cost increase linearly with the unit cost of filter aid while the optimum filtration rate and body feed rate both

Figure 38. Plots showing the effects of filter aid cost on optimum design conditions and total cost

 \sim

 $\sim 10^{11}$ km $^{-1}$

 $\mathcal{L}^{\text{max}}_{\text{max}}$

180

decrease. The effect of filter aid cost is much less than was expected. The optimum body feed rate decreased from 19 mg/1 to about 14 mg/1 and the total cost only increased from \$35/mg to about \$42/mg when the cost of filter aid was increased from \$60/ton to \$120/ton.

Effect of suspended solids concentration on optimum design conditions

For certain waters, the concentration of suspended solids in the filter influent will vary from season to season, day to day, or even from one hour to another. To study the effect of suspended solids concentration, the optimum design conditions for filtering an influent containing from 5 to 50 mg/1 of suspended solids were determined. The data contained in Table 12 were used along with the β predicted equation:

$$
\beta = 10^8 \, \text{C}_\text{S}^2 \, \text{C}_\text{F}^{-2}
$$

The optimum filtration rate decreased with increasing concentrations of suspended solids as shown in Figure 39. All other factors increased when suspended solids were increased. The proportional increase in the optimum body feed rate was less than the proportional increase in the concentration of suspended solids. In this example, C_S/C_F increased from 0.42 at $C_S = 5$ mg/1 to 1.25 at $C_S = 50$ mg/1.

It has been stated as a rule-of-thumb that for a particular water there is a certain ratio of C_g/C_p that should be maintained for optimum operation. The results shown here indicate that this is not true.

Effect of cost factors on optimum design conditions

The effect of the price of filter aid on optimum design conditions has previously been discussed. Other cost factors which will vary with time and location are first cost and power, labor and maintenance costs. To study the effect of these cost factors two cases were considered:

Case 1. The filtration of effluent from the lime-soda ash softening process. The β prediction equation for this water:

$$
\beta = 10^{10.20} \, \text{c}_\text{S}^{-1.43} \, \text{c}_\text{F}^{-3.29}
$$

was developed by Dillingham (27) from data collected at Lompoc, California. Celite 503 filter aid was used at a price of \$69/ton.

Case 2. The filtration of iron-bearing water. The S prediction equation for this water:

 $\beta = 10^{9.33} C_{\rm s}^{-1.95} C_{\rm F}^{-1.95}$

was developed from data collected by Hall (35) for the filtration of University tap water with ferrous sulfate added. Celite 503 was used at a price of \$100/ton.

Figure 39. Plots showing the effects of suspended solids concentration on optimum design conditions and total cost

 \bar{z}

 \mathbf{r}

These cases were considered because for Case 1 the optimum head loss is well below the practical limit of 150 ft. For Case 2 the optimum head loss is limited by the 150 ft practical limit. Optimum design calculations were made with the first cost and labor and maintenance cost the same as and twice the values given in Table 12 and with power costs of 1.5 $\frac{c}{kwh}$ and 3.0 $\frac{c}{kwh}$. These calculations were made at all possible combinations of first, power, and labor and maintenance costs so that 8 optimum design conditions were determined for each case. The incremental values of q, H_+ , and C_F were 0.1 gpm/sq ft, 5 ft, and 5 mg/1, respectively.

The results of these calculations are presented in Table 14 for Case 1 and in Table 15 for Case 2. In general doubling the first, power, and labor and maintenance costs does not cause a very large change in the optimum design conditions. Unless a very small incremental value for q, H_+ , or C_c were used, no change in the respective optimum would be observed. However, it can be concluded from these results that:

- A. Increasing the power cost
	- 1. Decreases the optimum q
	- 2. Decreases the optimum H_+
	- 3. Increases the optimum C_{F}
	- 4. Increases the total cost

First cost	LM -- Labor and maintenance -- $2LM$ cost					
Power $cost$ --------- 1.5 ϕ/kwh						
F	$(2.5, 55, 30; 26.1)^{a}$ $(2.8, 60, 30; 31.5)$					
2F.	$(3.1, 60, 30; 31.2)$ $(3.2, 65, 30; 36.4)$					
Power $cost$ -------- 3.0 ¢/kwh						
$\mathbf F$	$(2.0, 35, 30; 28.9)$ $(2.5, 35, 35; 34.5)$					
2F	$(2.8, 35, 35; 34.3)$ $(3.1, 40, 35; 39.7)$					
^a (Filtration rate, Head loss, Body feed; Unit Cost) gpm/sq ft ft $mq/1$ \$/MG Effect of first, power, and labor and maintenance Table 15. costs on the optimum design conditions for the filtration of iron bearing water						
First cost	LM -- Labor and maintenance -- $2LM$ cost					
	Power $cost$ ---------- $1.5 \frac{c}{kwh}$					
\mathbf{F}	$(0.7, 150, 40; 73.3)^{d}$ $(0.8, 150, 45; 88.2)$					
2F	$(0.9, 150, 45; 88.0)$ $(0.9, 150, 45; 101.7)$					
Power cost --------- 3.0 ¢/kwh						
F	$(0.7, 140, 40; 83.5)$ $(0.8, 150, 45; 98.5)$					
2F	$(0.9, 150, 45; 98.3)$ $(0.9, 150, 45; 112.1)$					

Table 14. Effect of first, power, and labor and maintenance costs on the optimum design conditions for the filtration of lime-soda ash process effluent

^d (Filtration rate, head loss, body feed; unit cost) gpm/sq ft ft mg/1 \$/MG $\mathcal{L}_{\mathbf{a}}$

- B. Increasing the first cost
	- 1. Increases the optimum q
	- 2. Increases the optimum H_t
	- 3. Increases the optimum C_{F}
	- 4. Increases the total cost
- C. Increasing labor and maintenance costs
	- 1. Increases the optimum q
	- 2. Increases the optimum H_t
	- 3. Increases the optimum C_{F}
	- 4. Increases the total cost

APPLICATIONS

Variable Water Quality Situation

Figures 16-19 (pp. 80-82) show how the temperature and turbidity of the water in the Des Moines River near Boone, Iowa, vary during the year. If a precoat filter was to be used to filter this water, the temperature and turbidity used for making the optimum design calculations should be chosen so that the annual cost of filtration will be minimized. To study how the annual cost of filtering water from the Des Moines River is affected by the design temperature and design turbidity, it was assumed that the filter cake resistance prediction equation:

 $\beta = 10^{7.80}$ (T/C_F)^{2.43}

 $\frac{1}{2} \frac{1}{2} \frac{$

would be applicable. This β prediction equation was developed with data gathered from filter runs made with raw water from the Missouri River at Council Bluffs, Iowa, using Hyflo Super-Cel as filter aid (Appendix A, Table 28). The plot of log β versus log T/C_F for these filter runs is shown in Figure 40. The head loss versus time curve became exponential, indicating that a compressible cake was formed, when the T/C_p ratio was greater than 1.5. Therefore, the minimum value of C_F used in all design calculations was chosen so that the maximum value of T/C_p considered would be less than or equal to 1.5. All optimum design and optimum operation calculations

Figure 40. Log β versus log T/C_F for raw water from the Missouri River at Council Bluffs, Iowa

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 \mathcal{A}

Hyflo Super-Cel filter aid

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1

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 T/C_F

 $\frac{1}{5.0}$

 $\frac{1}{10.0}$

 $\boldsymbol{\gamma}$.

 \mathfrak{a}

 $\frac{1}{0.1}$

were made with the ξ index, in-place bulk density and cost of Hyflo Super-Cel equal to 5.2 x 10^9 ft/lb, 20.7 lb/cu ft and \$90/ton, respectively. Other cost data are shown in Table **12.**

First of all, optimum design calculations were made with the design turbidity equal to the mean turbidity of 30 JTU and with the design temperature varied from 0 to 30° C. The results of these calculations are shown in Table 16. From these results it was concluded that the design temperature

Temperature		Optimum Filtration	Optimum Terminal	Optimum Body feed
(°C)	(°F)	rate (gpm/sq ft)	head loss (f _t)	rate (mg/1)
$\mathsf 0$	32	0.9	120	40
5	41	0.9	120	35
10	50	0.9	110	35
15	59	1.0	110	35
	68	1.0	105	35
$\begin{array}{c} 20 \\ 25 \end{array}$	77	1.0	105	30
30	86	1.0	100	30

Table 16. Effect of design temperature on optimum design conditions (Influent turbidity = 30 JTU)

does not have a very large effect on the optimum design conditions. Therefore, the main factor affecting the annual cost of filtering this water will be the design turbidity.

Next, calculations were made to determine the optimum design conditions at various design turbidities ranging from 5 to 100 JTU. The mean water temperature of 12° C was

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 $\mathcal{L}_{\rm{max}}$, $\mathcal{L}_{\rm{max}}$

used as the design temperature in all of these calculations. The results shown in Table 17 indicate the large effect the design turbidity has on the optimum design conditions.

Design Turbidity (JTU)	Optimum Filtration rate (gpm/sq ft)	Optimum Terminal head loss (f _t)	Optimum Body feed rate (mg/1)
5	1.9	45	10
10	1.6	60	20
20	1.2	90	30
30	1.0	115	35
40	0.9	140	40
50	0.8	150 (maximum	50
60	0.7	permissible 150	55
70	0.6	head loss) 150	60
80	0.6	150	65
90	0.5	150	70
100	0.5	150	80

Table 17. Effect of design turbidity on optimum design con-
ditions (Water temperature = 12° C) (Water temperature = 12° C)

Table 17 lists the optimum design conditions for a precoat filtration plant designed for continuous operation at each of several turbidities. An actual plant must be designed using only one set of these optimum conditions. In general, the plant will operate at turbidity levels different than that used in optimizing the plant design. In such a plant, the flow rate and head loss would be fixed in the design and only the body feed rate can then be reoptimized when the turbidity level changes.

Optimum operation calculations for various design

turbidities were then made with several sets of design data from Table 17 to determine the optimum body feed rate and minimum filtration cost for influent turbidities varying from 5 to 100 JTU. For example, if the design turbidity is 30 JTU, the filter would be designed to operate at a filtration rate of 1.0 gpm/sq ft and with a terminal head loss equal to 115 ft (Table 17). Optimum operation calculations for several other levels of turbidity were-made, therefore, with the filtration rate equal to 1.0 gpm/sq ft and the terminal head loss equal to 115 ft. The results for this example are shown in Table 18.

Influent turbdiity (JTU) Optimum Operating Body feed rate (mg/L) Unit cost (\$/MG) 5 **10 20** 30 40 50 **60** 70 **80** 90 **100** 5 10 25 35 50 **60** 75 90 105 120 135 38.5 **42.3** 50.0 58.0 66.5 75.2 84.4 93.9 103. 8 114.1 124.7

Table 18. Optimum operating conditions at various influent turbidities (Design turbidity = 30 JTU and design temperature = 12° C)

The effect of the design turbidity on the filtration cost at various influent turbidities is shown in Figure 41. As the difference between the influent turbidity and the design turbidity increases, the difference between the filtration costs for optimum operation and optimum design also increases. If the design turbidity is very low, the difference between the filtration costs for optimum operation and optimum design are very large at high turbidities. This is due to the extremely short filter run lengths which result when a filter is designed for a low turbidity but operated at a high turbidity.

The annual cost of filtration was calculated for each design turbidity. An example of how annual cost calculations were made is shown in Table 19. The number of weeks shown in column 2 of Table 19 were determined from the frequency distribution diagram shown in Figure 19. For example, the turbidity was between 0 and 5 JTU in 13 of the weekly samples, between 5 and 10 JTU in 8 of the weekly samples, etc. It was assumed that the turbidity was 5 JTU for 13 weeks, 10 JTU for 8 weeks, etc. This assumption causes the calculated annual costs to be larger than the actual cost would be if optimum operating conditions were maintained at all times. However, in an actual situation it is doubtful that optimum operating conditions would be maintained at all times.

Figure 41. Plots showing the effects of design turbidity on the unit cost of filtration at various influent turbidities

Figure 42 is a plot of the annual unit cost of filtration versus the design turbidity. The shape of the frequency distribution diagram for turbidity (Figure 19) might lead one to predict that the optimum design turbidity Figure 42. Annual unit cost of filtration versus the design turbidity

> Raw water from the Des Moines River near Boone, Iowa

> > l.

 \sim

would be less than the mean turbidity of 30 JTU. However, from Figure 42 the optimum design turbidity is about 40 JTU. The optimum design turbidity is higher than might be expected because of the short filter runs and high costs that result when a filter designed for a low turbidity is operated at a high turbidity. Another observation from Figure 42 is that using a design turbidity that is too low has a more adverse effect than using a design turbidity that is larger than the optimum.

Pretreatment of Raw Water

It would be economical to pretreat a water prior to filtering if the cost of filtering the pretreated water plus the cost of pretreatment is less than the cost of filtering the raw water. Precoat filtration data have been collected using Celite 535 filter aid with both raw and pretreated water from an impounding reservoir at Albia, Iowa. All of the filter runs were made using a U.S. Army mobile water treatment unit. The pressure filter in this unit contains 3.5 - inch diameter septa which comprise a total surface area of 10 sq ft. The filter is described in detail by Bridges (15).

The data obtained with pretreated water have been reported previously (15) and are summarized in Appendix A (Table 27). The water was pretreated in the city treatment

plant. Pretreatment consisted of coagulation and settling in a solids contact type upflow clarifier with coagulant dosages of 26 mg/1 of alum and 26 mg/1 of lime when the raw water turbidity was approximately 10 JTU. Due to carryover of floe from the clarifier, the turbidity of the pretreated water averaged approximately 6 JTU. Data obtained with the raw water were collected when the average turbidity was 10 JTU (Appendix A, Table 28, filter runs 1-10).

 β prediction equations were determined to be:

$$
\beta = 10^{9.27} (\text{T/C}_{\text{F}})^{1.74}
$$

for the pretreated water, and:

$$
\beta = 10^{8.75} (\text{T}/\text{C}_{\text{F}})^{1.88}
$$

for the raw water. At the same ratio of turbidity to body feed rate, the filter cake resistance for the pretreated water was 4 to 6 times the filter cake resistance for the raw water. Thus, for this water, the value of pretreatment is to reduce the turbidity of the filter influent.

The optimum design conditions and filtration cost were computed for both the raw and pretreated water at various influent turbidities. These results are listed in Table 20. All calculations were made with the ξ index, in-place bulk density and cost of Celite 535 equal to 1.9 x 10^9 ft/lb, 19.9 lb/cu ft and \$98 per ton, respectively. Other design

data are shown in Table 12. The total cost of pretreatment was determined to be \$15.4 per MG. This figure was calculated using the following cost data:

lime \ldots \ldots $\frac{22}{2}$ per ton alum \cdots \cdots $\ddot{}$ \$80 per ton clarifier \ldots \ldots \$26,500 first cost

From the unit cost figures listed in Table 20, it is evident that if the raw water turbidity was 10 JTU, it would not be economical to pretreat the water unless the turbidity

Table 20. Optimum design conditions for the filtration of raw and pretreated water at Albia, Iowa

aIncludes the cost of pretreatment.

of the pretreated water was 2 JTU or less. It is assumed that an acceptable filter effluent is obtained with either raw or pretreated water. The turbidity of the pretreated water from the city treatment plant was high (6 JTU) because of floe carryover caused by intermittent operation of the clarifier. Jar tests with the raw water showed that if the clarifier was operated properly, the turbidity could be reduced to less than 1.5 JTU using 20 to 30 mg/1 of alum.

If the turbidity of the raw water was 30 JTU, it would be economical to pretreat the water even if the turbidity of the pretreated water was greater than 6 JTU (Table 20).

Backwash Waste Disposal

The method chosen for disposal of the backwash waste from a precoat filtration plant will depend upon local conditions such as availability and cost of land for dewatering and land fill facilities, the sewage treatment system and method of sludge disposal, the loading that the backwash waste would present on the sewage collection and treatment facilities, etc. The spent filter aid from swimming pool filters is often discharged directly to the sewer. Since this represents only a small proportion of the total sewage flow, no significant problems in the sewage collection or

treatment systems have been observed (lb, p. 6). If the backwash waste represents a sizable loading on the sewage collection and treatment systems, problems may result from clogging of the sewers and abrasion of pumps and other mechanical equipment. If sewage sludge is treated by anaeroerobic digestion, the spent filter aid will occupy digestion tank space needed for organic materials, thus reducing digestion efficiency. However, if the sewage sludge is dewatered by vacuum filtration, the spent filter aid may be beneficial for increasing the porosity of the sludge cake.

In general, it is recommended that backwash wastes be dewatered in a settling lagoon or tank and then disposed of by land fill. This method is used by most of the existing precoat filtration plants. Since waste filter aid has a tendency to shrink under loading, no land fill site should be contemplated for building purposes unless the deposits are compacted in a controlled manner (lb, p. 10).

SUMMARY AND CONCLUSIONS

The long-term goal of the research in precoat filtration at Iowa State University has been to provide a scientific basis for the optimum design of precoat filters for municipal applications. To realize this goal it was necessary to develop :

1. A theory to predict filter performance in terms of filter run constants (filtration rate, water temperature, etc.) and the filtrability characteristics of the filter influent.

2. The means of predicting the filtrability characteristics of the filter influent.

3. A method of employing the theory to optimize filter design.

The theory of precoat filtration and the methods for predicting filter cake resistance and optimizing filter design were developed in studies made by LaFrenz (43) and Dillingham (27). Further studies have been made to determine the applicability of the theory for the filtration of water containing various types of suspended solids, to determine the characteristics of different types and grades of filter aids, and to determine the applicability of the method of predicting filter cake resistance for these waters and filter aids.

The objectives of this study were basically two-fold. The first and primary objective was to outline the procedures for determining the optimum design conditions of a proposed precoat filtration plant. These procedures include collection of filtration and cost data, reduction of the data and development of equations for predicting filter cake resistance, and calculation of the optimum design conditions.

The second objective was to review and summarize the research on which the method of optimizing the design of precoat filters is based. The goals of this review were to define the limitations of the present theory of precoat filtration and to determine what improvements could be made in the theory and method of predicting filter cake resistance.

Based on the review of previous research and the results presented in this study, the following significant conclusions can be made.

1. The filtration data necessary for optimizing the design and/or operation of a precoat filtration plant can be collected using a small-scale, constant-rate filter (SSCR filter). Results from this study indicate that the SSCR filter may be used to determine both the filtrability characteristics of the filter influent and the quality of the filter effluent.

2. The filter cake resistance indicated by the β index can best be predicted by the equation:
$$
\beta = 10^{b_1} \, c_S^{b_2} \, c_F^{b_3} \tag{33}
$$

To develop such an equation to predict β for filtering a particular water, it is necessary to collect filtration data with significant variation in both the suspended solids concentration and the rate of body feed. If the suspended solids concentration of the filter influent is expected to remain constant at the value for which the filtration data is collected, the equation:

$$
\beta = 10^{b_1} \, \text{C}_{\text{F}}^{b_2} \tag{27}
$$

is applicable. The g prediction equation:

$$
\beta = 10^{b_1} (C_S/C_F)^{b_2}
$$
 (15)

is valid only under the assumption (which is frequently invalid) that the value of β is the same for equal ratios of suspended solids concentration to body feed concentration (i.e. no concentration effects).

3. The apparent detention time observed during the initial stages of a precoat filter run is due to both initial dilution of the filter influent in the filter housing and the time required for a filter cake to form. From the results gathered in this study, the apparent detention time is inversely proportional to the concentration of suspended solids in the filter influent and has little, if any, dependence on

the amount of body feed. Theoretically, the apparent detention time is also inversely related to the filtration rate.

4. The apparent detention times observed in this study have no significant effect on the calculated optimum design conditions and can therefore be ignored when optimum design calculations are made.

5. The theory of precoat filtration is limited to filtration through incompressible filter cakes. Therefore, optimum design calculations are limited by the lowest body feed rate which results in the formation of an incompressible filter cake.

6. The main factors which determine the optimum design conditions of a precoat filter are the concentration and filtrability characteristics of the suspended solids in the filter influent. Therefore, the determining factor of the optimum design conditions is the design water quality and the main criteria for selecting the type and grade of filter aid is the quality of filter effluent that is produced.

7. The digital computer is a valuable tool for reducing filtration data and calculating the optimum design conditions for precoat filtration plants. However, all of the computations necessary for design a precoat filtration plant can be done manually in a reasonable time period.

8. Pretreating a water by coagulating and settling may

increase the specific resistance of the filter cake formed during precoat filtration. However, it may still be economical to pretreat the water if the suspended solids concentration of the pretreated water is significantly less than the suspended solids concentration of the raw water.

RECOMMENDATIONS

Based on the results of this and previous studies it is recommended that the procedures outlined in this dissertation be used to optimize the design of proposed precoat filtration plants and to optimize the operation of precoat filtration plants now in operation. It is further recommended that :

1. An investigation be undertaken to determine the ability of the SSCR filter to predict filter cake resistance and effluent quality at several precoat filter installations now in operation. The collection of accurate filtration data at the proposed plant site is an important prerequisite for the design of a precoat filtration plant and an investigation of this type would definitely prove the value of the SSCR filter for collecting the required filtration data.

2. The effects of suspended solids concentration, body feed rate, filtration rate, and filter aid grade on apparent detention time should be studied in more detail. It is suggested that filter runs be made using the SSCR filter and a suspension known to exhibit a wide range of apparent detention times with different suspended solids concentrations. Several series of filter runs could be made with C_g , C_F , q, or the grade of filter aid as the only variable to determine the effect of each individual variable on the apparent

detention time.

3. A study be made to determine if the filtrability characteristics, as indicated by the 6 prediction equation, of surface waters show annual variations.

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APPENDIX A

Summary of Precoat Filtration Data

A summary of the data from almost all of the precoat filter runs made by researchers at Iowa State University are contained in this appendix. The data are identified by:

- 1) Name of the researcher
- 2) Filter used
- 3) Water filtered
- 4) Dates, inclusive, during which the filter runs were made

The filter aid identification system used is listed in Table 21. Different bags of the same filter aid are indicated

Manufacturer	Grade of filter aid	Identification system designation
Sil-Flo Corp.	272^{a} 332 ^a 443^{a}	S ₂ S ₃ S ₄
Johns-Manville Products Corp.	Hyflo Super-Cel Celite 503 Celite 535 Celite 545	HFC J0 J3 J ₄
Eagle-Picher Industries, Inc.	$FW-60$ $FW-50$ $FW-20$	E6 E5 E2
Great Lakes Carbon Corp. (Dicalite)	4200 Speedex	G ₄ G1

Table 21. Filter aid identification system

^aPerlite filter aids. All other filter aids are diatomite.

by a bag identification number attached to the filter aid designation. For example, a filter aid identified with designation J3-12 would be a filter aid taken from bag 12 of a Celite 535 shipment.

Table 22. LaFrenz's pilot plant data (43)

 $\frac{1}{2} \left(\frac{1}{2} \right) \right) - \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \right) - \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \right) - \frac{1}{2} \$

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Table 23 (Continued)

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Table 23 (Continued)

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Table 23 (Continued) ------------

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Table 24. Hall and Hawley's pilot plant data (35, 37)

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Table 24 (Continued)

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Table 24 (Continued)

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Table 25. Regunathan's pilot plant data (61)

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Table 25 (Continued)

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Table 26. Dillingham's Lompoc, California, data (27)

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Table 26 (Continued) _____________

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{i=1}^n\frac$

Table 26 (Continued)

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Table 27. Bridges' data from filtration of pretreated surface waters (15) -----------------

 $\label{eq:2.1} \frac{1}{2}\sum_{i=1}^n\frac{1}{2}\left(\frac{1}{2}\sum_{i=1}^n\frac{1}{2}\sum_{i=1}^n\frac{1}{2}\sum_{i=1}^n\frac{1}{2}\sum_{i=1}^n\frac{1}{2}\sum_{i=1}^n\frac{1}{2}\sum_{i=1}^n\frac{1}{2}\sum_{i=1}^n\frac{1}{2}\sum_{i=1}^n\frac{1}{2}\sum_{i=1}^n\frac{1}{2}\sum_{i=1}^n\frac{1}{2}\sum_{i=1}^n\frac{1}{2}\sum_{i=1}^n\frac{1}{2}\sum$

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Table 27 (Continued)

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Table 27 (Continued)

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Table 28. Bridges' data from filtration of raw surface waters

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Table 28 (Continued)

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 $\label{eq:2.1} \frac{d\mathbf{y}}{d\mathbf{x}} = \frac{1}{2} \sum_{i=1}^n \frac{d\mathbf{y}}{d\mathbf{x}} \mathbf{y}_i \mathbf{y}_i \mathbf{y}_i$

Table 28 (Continued)

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Table 28 (Continued)

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FILTER RUN NO	FILTER ATC	۰ GPM/SQ FT	INFLUENT TEMP DEG C	SUSPENDED SOLIDS INFLUENT MG/L	EFFLUENT JTU	BCCY FEED MG/L	BETA INDEX 4 -2 10 FT	R 0/0	COMMENTS
CERWICK		LABORATORY PILCT PLANT					UNIVERSITY TAP WATER PLUS KENTUCKY BALL CLAY (KAOLINITE)		SUMMER, 1967
1 A	HE C	1.0	25.1	17.4	0.04	42.4	972.1	99.668	
18	HEC	1.9	25.1	17.4	0.04	42.4	123373.	99.707	72.4 MG/L ALUM FLASH MIXED WITH FILTER INFLUENT
24	HE C	1.9	25.1	17.4	0.02	44.8	792.3	99.900	
26	HEC.	1.0	25.1	17.4	0.02	44.8	71684.	99.911	42.1 MG/L ALUM
34	HFC.	1.0	$2 - 2$	19.8	$0 - 00$	78.7	2204.8	99.928	
35	HF C	1.0	25.2	19.8	0.00	38.7	96526.	59.992	31.6 MG/L ALUM
44	HEC.	1.0	25.3	19.8	0.01	35.I	2602.4	55.809	
48	HEC	1.0	25.3	19.8	0.01	35.1	26970.	99.999	15.0 MG/L ALUM
κ.	HE C	1.0	25.7	\cdots	---	$\qquad \qquad \cdots$	$\qquad \qquad \cdots$	$\qquad \qquad \cdots$	VOID ** MECHANICAL DIFFICULTIES
64	HEC	1.0	25.2	18.7	0.01	42.4	1212.8	99.880	
68	HFC	1.0	25.2	18.7	0.01	42.4	4455.2	99.988	6.78 MG/L ALUM
78	HE C	1.9	25.0	19.5	0.01	39.2	2270.2	99.970	
7R	HFC	1.0	25.0	19.5	0.01	39.2	38364.	99.941	23.4 MG/L ALUM
98	HEC	1.0	24.8	19.5	0.01	42.9	1578.8	99.628	
RB.	HFC	1.0	24.8	18.5	0.01	42.8	120856.	99.995	58.1 MG/L ALUM
92	HFC	1.0	25. C	19, 7	0.03	41.6	1912.8	99.836	
QR.	HF C	1.2	25.0	19.7	0.03	41.6	179515.	59.740	120. MG/L ALUM
104	HE C	1.0	25.2	19.5	0.00	38.7	2311.9	59.761	
10 ^m	HEC	1.0	25.2	19.5	0.00	38.7	11781.	99.938	50.1 MG/L ALUM
114	HEC	1.2	25.2	56.7	0.00	69.4	2634.3	99.940	
118	HEC	1.9	25.2	56.7	0.00	69.4	57146.	99.896	57.3 MG/L ALUM
12A	HEC	1.9	24.6	58.6	0.02	132.0	944.8	99.835	
129	HEC	1.0	24.6	58.6	0.02	132.0	6204.2	99.960	18.7 MG/L ALUM
134	HEC	1.0	24.9	57.5	0.00	130.5	993.5	99.926	
139	HEC	1.0	24.9	57.5	0.00	130.5	2321.1	99.898	5.5 MG/L ALUM
14A	HFC	1.0	24.7	54.6	0.00	141.0	790.1	99.966	
148	HEC	1.0	24.7	54.6	0.00	141.0	3210.7	99.935	11.4 MG/L ALUM
154	HEC	1.0	25.0	57.8	0.01	129.5	989.1	99.956	
159	re C	1.2	25.0	57.8	0.01	129.5 129.0	13677. 641.8	99.958 59.939	36.9 MG/L ALUM
164	HFC.	1.0	24.9	51.6	0.00			99.992	
169 17A	HEC. HEC	1.0	24.9 25.0	-1.6	0.00 0.01	129.0 119.0	11296. 1039.7	99.955	46.1 MG/L ALUM
176	HF C	1.0 1.0	25.0	54.5 54.5	0.01	119.0	13028.	99.968	30.3 MG/L ALUM
194	HFC	1.0	24.7	55.1	0.00	137.0	759.5	99.956	
1 PR	HEC.	1.0	24.7	55.1	0.00	137.0	21901.	99.901	63.6 MG/L ALUM
194	HFC.	1.0	25.0	54.9	0.00	137.0	876.0	99.813	
198	HEC	1.0	25.0	54.9	0.00	137.0	28420.	99.382	90.4 MG/L ALUM
70A	HFC	1.9	25.0	57.0	0.00	134.5	929.3	99.935	
298	HFC.	1.0	25.0	57.0	0.00	134.5	1595.3	99.985	2.41 MG/L ALUM
21 A	HF C	1.0	24.0	57.0	$\qquad \qquad \cdots$	132.2	766.6	99.752	
215	HF C	1.0	24.0	57.0	$---$	132.2	920.7	99.947	O.O MG/L ALUM
22 A	HEC	1.9	25.0	19.7	---	42.4	1151.1	99.865	
228	HEC	1.2	25.0	19.7	---	42,4	987.8	99.953	0.0 MG/L ALUM

Table 29. Cerwick's pilot plant data (20)

FILTER RUN NO	FILTER AI D	$\mathbf Q$	INFLUENT TEMP		SUSPENDED SOLIDS INFLUENT EFFLUENT	BCCY FEED	BETA INDEX $4 -2$	R	COMMENTS
		GPM/SQ FT	DEG F	MG/L	MG/L	MG/L	10 FT	c/a	
ARORA		LABORATORY PILOT PLANT				UNIVERSITY TAP WATER PLUS FERROUS SULFATE		12/24/66 - 3/12/67	
$\mathbf{1}$	JO.	1.0	60	6.69	0.06	52.4	1920.		99.562 TRIAL RUN
2	JO.	$1 - 0$	60	3.56	0.03	95.	$\qquad \qquad \cdots$	$\frac{1}{2}$	TRIAL RUN ** BODY FEED STOPPED IN MIDDLE OF RUN
3	JO.	1.0	60	8.34	0.00	86.	2931.		99.806 TRIAL RUN BODY FEED STOPPED DURING RUN
4	JO.	1.0	60	8.43	0.23	96.5	2259.	99.884	
5	\$3	$1 - 0$	60	8.31	0.07	84.5	1346.	99.916	
6	S3	1.0	60	8,55	0.05	91.5	1253.	99.746	
7	53	1.0	60	8.89	0.03	91.	946.	99.975	
8	S3	1.0	60	8.51	0.03	58.5	2080.	99.662	
9	S3	$1 - 0$	60	8.52	0.07	112.	705.6	99.986	
10	S3	$1 - 0$	60	8.36	0.11	35.4	7631.	99.816	
$\mathbf{11}$	S ₃	1.0	60	8.21	0.15	23.1	15470.	99.150	
12	S3	1.0	60	11.41	0.10	149.0	$- - -$	$\frac{1}{2}$	CONCENTRATION OF IRON IN INFLUENT VARIED DURING RUN
13	S3	1.0	60	8.95	0.03	227.0	280.0	99.931	
14	S3	1.0	60	8.71	0.04	352.0	163.6	99.929	
15	\$3	1.0	60	$---$	$\frac{1}{2}$	$- - -$	$- - -$	$\qquad \qquad -\qquad -$	RUN DISCONTINUED ** BYPASS VALVE LEFT OPEN
ARDRA		LABORATORY PILOT PLANT				UNIVERSITY TAP WATER PLUS FERRIC CHLORIDE		$3/15/67 - 3/4/68$	
15A	\$3	1.0	60	6.90	\cdots	74.3	$\qquad \qquad \cdots$	$\frac{1}{2} \left(\frac{1}{2} \right) \left(\frac$	EXPONENTIAL HEAD LOSS CURVE
16	53.	$1 - 0$	60.	7.48	0.06	108.0	8103.	99,778	
17	53.	$1 - 0$	60	7.43	0.06	219.5	1747.	99.820	
18	$52 - 22$	1.0	60	6.91	0.12	150.0	2453.	99.937	
19	$52 - 22$	1.0	60	7.51	0.05	211.2	1553.	99.946	
20	$S2 - 22$	1.0	60	7.45	0.05	249.3	1166.	99.979	
21	$S2 - 22$	1.0	60	7.52	0.06	322.6	825.1	99.965	
22	$52 - 22$	$1 - 0$	e e	7.25	0.04	139.5	3291.	99.846	
23	$S2 - 22$	1.0	60	7.15	0.06	125.6	3518.		99.861 DUPLICATION OF RUN 22
24	$52 - 22$	1.0	60	6.20	0.03	117.2	3245.		99.872 DUPLICATION OF RUN 22
25	$52 - 22$	1.0	60	6.51	0.03	160.8	2135.	99.881	
26	$52 - 22$	1.0	60	6.85	0.06	242.6	1169.	99.956	
27	$52 - 22$	1.0	60	6.98	0.04	282.2	867.4	99.979	
28	$52 - 22$	$1 - 0$	60	6.70	0.06	73.5	10920.	99.878	
29	$53 - 4$	1.0	60	7.50	0.09	124.0	5453.	99.784	
30	$53 - 4$	1.0	60	7.21	0.07	125.4	5305.		99.846 DUPLICATION OF RUN 29
31	$53 - 4$	$1 - 0$	60	7.07	0.07	118.8	5906.		99.903 DUPLICATION OF RUN 29
32	$53 - 4$	1.0	60	7.11	0.07	96.5	10730.	99.890	
33	$53 - 4$	1.0	60	7.21	0.06	68.9	23310.	99.481	

Table 30. Arora's pilot plant data (2)

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Table 30 (Continued)

FILTER RUN NO	FILTER AID.	۰ GPM/SQ FT	INFLUENT TEMP DEG F	SUSPENDED SOLIDS INFLUENT EFFLUENT		BCDY FEED	BETA INDEX $4 - 2$	R	COMMENTS
				MG/L	MG/L MG/L	10 FT	0/0		
34	$53 - 4$	1.0	60	7.09	0.04	247.9	1429.	99.936	
35	$53 - 4$	1.2	60	7.36	0.04	180.6	2706.	99.736	
36.	$52 - 4$	1.2	60	8.00	0.06	311.4	1134.	99.920	
37	$53 - 4$	1.9	60	8.47	0.16	341.6	1004.	99.932	
3A	$53 - 4$	1.2	60	7.62	0.08	412.3	650.8	99.951	
٦q	$52 - 4$	1.0	€C.	7.54	0.05	170.0	2564.	99.910	
49	$52 - 4$	1.0	6C	7.53	0.03	160.0	1750.	99.849	
41	$52 - 4$	1.2	60	7.62	0.03	262.4	1047.	99.957	
47	$52 - 4$	1.0	t0	7.17	0.03	309.4	728.7	99.936	
43	$52 - 4$	1.0	60	7.44	0.03	7C9.0	723.3		99.925 SAME AS RUN 42 EXCEPT NC COPPER SULFATE ADDED
44	$52 - 4$	1.9	6C	7.37	0.09	51.2	36670.	98.003	
45	$52 - 4$	1.0	60	7.24	0.07	68.5	18290.	59.594	
46	$52 - 3$	1.0	60	7.60	0.05	66.8	21600.	99.567	
47	$5 - 2$	1.2	60	6.69	0.08	50.7	34580.	97.904	HEAD LOSS CURVE ALMOST EXPONENTIAL
4 A	$52 - 7$	1.9	÷С	7.2 ₆	0.12	117.6	4949.	99.699	
49	$52 - 3$	1.0	60	9,52	0.17	166.7	3827.	95.817	
∙۰	$52 - 3$	1.0	60	8.96	0.11	150.6	2205.	99.900	
51	$52 - 3$	1.9	6C	7.03	0.13	257.4	1263.	99.916	
52.	$52 - 3$	1.0	60	8.20	0.07	292.1	1035.	99.972	
53.	$52 - 1$	1.0	60	6.95	0.07	130.8	4064.	99.602	
54	$52 - 1$	1.0	60	7.21	0.06	155.1	2759.	99.873	
55.	$52 - 1$	1.0	60	7.31	0.06	200.5	1822.	99.962	
-6	$52 - 1$	$1 - 0$	60	8.02	0.06	207.9	1728.	99.676	
57.	$52 - 1$	1.0	60	8.08	0.08	301.1	890.5	99.980	
5 Q	$52 - 1$	1.0	6C	$\qquad \qquad - -$	0.12	---	$\frac{1}{2}$	$\frac{1}{2} \left(\frac{1}{2} \right) \left(\frac$	CONCENTRATION OF IRON IN INFLUENT VARIED DURING RUN
ç٥	$52 - 1$	1.0	6C.	7.96	0.15	98.2	9138.	99.647	
60	$52 - 1$	1.0	60.	7.87	0.11	101.4	8736.		99.927 DUPLICATION OF RUN 59
61	$52 - 20$	1.0	€C.	7.79	0.06	92.8	9820.	99.841	
62	$52 - 20$	1.2	60	7.94	0.07	78.4	16590.	99.642	
63	$52 - 20$	1.7	62	7.84	0.05	102.7	6576.	95.862	
64	$52 - 20$	1.0	60	7.99	0.07	127.1	5208.	99.936	
65	$52 - 20$	1.0	60	7.93	0.06	148.3	3576.	99.935	
66	$52 - 20$	1.0	60	7.91	$0 - 05$	252,0	1311.	99.966	
e7	$52 - 22$	1.9	60	8.27	0.06	86.6	14010.	99.506	
68	$52 - 23$	1.0	60	P.00	0.07	123.6	6271.	99.796	
69	$52 - 23$	1.0	60	7.59	0.04	125.4	5400.	99.475	
70	$52 - 23$	1.0	60	7.63	0.07	188.6	2213.	59.851	
71	$52 - 23$	1.0	60	8.32	0.17	242.1	1734.	99.960	
72	$52 - 23$	1.0	60	7.58	0.04	70.1	16980.	99.762	

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Table 30 (Continued)

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Table 30 (Continued)

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Table 30 (Continued)

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Table 30 (Continued)

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Tablo 31. Bridges' data from filtration of trickling filter effluent

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Table 31 (Continued)

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Table 32. Bridges and Arora's SSCR filter data

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Table 32 (Continued)

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Table 32 (Continued)

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APPENDIX B

Derivation of Precoat Filtration Equations

Basic equations

The precoat filtration equations are derived from the modified Darcy equation which was previously given as:

$$
\frac{\mathrm{d}V}{\mathrm{Ad}t} = \frac{K_1}{\mu} \frac{\mathrm{d}P}{\mathrm{d}L} \tag{3}
$$

where :

- V = volume of filtrate passing through the bed in time t (L^3)
- A = gross cross-sectional area of the porous media perpendicular to the direction of flow $[L^2]$
- K_1 = modified permeability coefficient independent of viscosity $[L^2]$

$$
\mu = \text{dynamic or absolute viscosity [FTL}^{-2}]
$$

 dP/dL = pressure gradient [FL⁻³]

- $P = pressure$ loss across the porous media in the direction of flow $[FL^{-2}]$
- $L =$ thickness of the porous media in the direction of flow [L]

The use of the modified Darcy equation requires that the following assumptions be made:

Assumption 1: Enough body feed is added to form an essentially incompressible filter cake.

Assumption 2; The flow through the bed is laminar. Equation 3 can be changed to:

$$
v = gi/va \tag{51}
$$

since :

$$
v = (1/A) \, \text{d}V/\text{d}t = \text{approach or face velocity } [LT^{-1}]
$$

i = $(\text{d}P/\text{d}L)\gamma_w = \text{d}H/\text{d}L = \text{hyd}r \text{a} \text{l} \text{i} \text{c} \text{gradient } [-1]$
v = $\mu g/\gamma_w = \text{kinematic viscosity } [L^2T^{-1}]$

and

a = specific resistance based on the volume of filter media $[L^{-2}]$

where :

$$
\gamma_{\text{w}} = \text{density of water } [FL^{-3}]
$$

g = gravity constant $[LT^{-2}]$
H = head loss or pressure difference in terms of height
of a water column [L]

Equation 51 can be applied to filtration through precoat filters by applying it separately to the precoat and filter cake. The derivation is simplified if constant rate conditions are imposed.

Assumption 3: Constant rate filtration.

Head loss through precoat - any septa

When cylindrical septa are used, the outer surface area of the precoat is slightly larger than the outer area of the

septa. However, because the precoat is very thin, the following assumption may be made:

Assumption 4: The outer surface area of the precoat layer is approximately equal to the outer surface area of the septa.

Therefore, Equation 51 can be written for the precoat as

$$
q = \frac{\nu a_p}{g} \frac{L}{h} \frac{d}{h}
$$

since :

 $q = v = Q/A_s$ = flow rate per unit septum area of filtration rate $[LT^{-1}]$ $Q =$ flow rate $[L^3T^{-1}]$ A_{s} = septum area [L²]

and the subscript p refers to the precoat.

Rearranging and substituting V_p/A_g for the thickness of the precoat, then

$$
H_p = \frac{qv}{g} \cdot \frac{a_p V}{A_s}
$$

where :

$$
V_p = \text{volume of precout } [L^3]
$$

If the specific resistance is based on the weight of filter aid in the precoat rather than the volume of filter aid then

$$
H_D = q \nu \xi w / g \tag{5}
$$

where :

- ξ = filter aid resistance index or ξ index = specific resistance of clean filter aid based on the weight of filter aid $[IF^{-1}]$
- w = weight of filter aid in the precoat per unit septum area $[FL^{-2}]$

Equation 5 is valid for any type of septum as long as the precoat is thin.

Head loss through filter cake - cylindrical septa

If cylindrical septa are used, the surface area of the filter cake increases during a filter run and therefore the face velocity, v, decreases when a constant rate of flow is maintained. Since v is directly proportional to i, the hydraulic gradient across a cylindrical filter cake is not constant throughout the cake. Therefore, to apply Equation 51 to a cylindrical filter cake, the hydraulic gradient must be expressed in differential form and Equation 51 may be written as

en as

$$
v = \frac{g}{\nu a_c} \frac{dH}{dL_c}
$$

or

 $dH_C = \frac{VV}{g} a_C dL_C$ (52)

where the subscript c refers to the filter cake.

The desired equation is one which equates head loss to

an easily measured variable such as time. To convert Equation 52 to terms of time, t, in place of filter cake thickness, L_{α} , consider a cylindrical septum with radius R_{α} . The small volume of filter cake formed during the interval of time dt is:

$$
dV_c = Q\gamma_w S_f dt / \gamma_c \tag{53}
$$

where :

- dv_c = volume of filter cake formed in the time interval dt $[L^3]$
	- S_f = weight fraction of combined solids + body feed in the water in the filter housing $[--]$

At the end of the precoating operation the filter housing is full of clean water. Therefore, during the filtering operation S_f is less than S_i (weight fraction of combined solids + body feed in the filter influent) because of the effect of initial dilution. But, S_f can be written in terms of s_i if the following assumptions are made:

Assumption 5: The filter housing is a completely mixed system.

Assumption 6; No suspended solids or body feed pass through the filter cake.

Drawing a mass diagram for the filter:

Thus, in the time increment Δt

$$
\texttt{Weight of solids entering} = \texttt{QY.S}, \Delta t
$$

and

Weight of solids removed =
$$
Q\gamma_{w}S_{f}\Delta t
$$
.

The change in weight of combined solids + body feed in suspension in the filter is therefore:

$$
\Delta W = Q\gamma_{\rm w}(S_{\rm j} - S_{\rm f}) \Delta t
$$

Dividing through by the weight of water in the filter housing yields :

$$
\frac{\Delta W}{V_f \gamma_w} = \frac{Q \gamma_w (S_i - S_f) \Delta t}{V_f \gamma_w}
$$

or

$$
\Delta S_f = \delta (S_i - S_f) \Delta t
$$

where :

$$
\Delta S_{f} = \Delta W/V_{f} \gamma_{w}
$$
\n
$$
\delta = Q/V_{f} = \text{theoretical dilution rate} [T^{-1}]
$$
\n
$$
V_{f} = \text{volume of filter housing} [L^{3}]
$$

Passing to the limit leads to a differential equation that can be integrated:

$$
\int \frac{dS_f}{S_i - S_f} = \int \delta dt
$$

Therefore,

$$
\ln (S_i - S_f) = -\delta t + C
$$

and

$$
S_i - S_f = e^{-\delta t} e^C
$$

where :

 $C =$ integration constant

For the initial condition $S_f = 0$ at t = 0, then $e^C = S^2$ and

$$
S_{f} = S_{i} - S_{i}e^{-\delta t}
$$

= $S_{i} (1 - e^{-\delta t})$
= $(C_{S} + C_{F}) 10^{-6} (1 - e^{-\delta t})$ (54)

since :

 \cdot

$$
S_{\text{i}} = (C_{\text{S}} + C_{\text{F}}) 10^{6}
$$

where :

 C_S = concentration of suspended solids in influent [--] C_F = concentration of body feed in influent [--]

Substitution for S_f in Equation 53 yields:

$$
dV_C = \frac{Q\gamma_W}{\gamma_C} (C_S + C_F) 10^{-6} (1 - e^{-\delta t}) dt
$$
 (55)

It is then necessary to make the following assumption:

Assumption 7: The solids removed in the filter cake do not increase the cake thickness appreciably over the thickness that would result if the cake caontained only body feed.

This is equivalent to the expression:

$$
\frac{C_F}{\gamma_p} \approx \frac{C_S + C_F}{\gamma_c}
$$

The terms in the above expression are assumed to remain constant with time.

Assumption 8: γ_p and γ_c remain constant throughout a filter run.

Assumption 9: C_S and C_F remain constant throughout a filter run.

Substitution for
$$
(C_S + C_D) / \gamma_C
$$
 in Equation 55 leads to:
\n
$$
dV_C = \frac{Q\gamma_W}{\gamma_P} C_F (10^{-6}) (1 - e^{-\delta t}) dt
$$
\n(56)

and since $dL_c = dV_c/A$, substitution for dL_c in Equation 52 yields the differential equation for precoat filtration:

$$
dH_C = \frac{v_v}{g} a_C \left[\frac{Q\gamma_w}{A\gamma_p} C_F \right] (10^{-6}) (1-e^{-\delta t}) dt
$$

$$
dH_C = \frac{v^2 v}{g} \left[\frac{a_C \gamma_w}{\gamma_p} (10^{-6}) \right] C_F (1-e^{-\delta t}) dt
$$

$$
dH_{\rm c} = \frac{v^2 v}{g} \beta C_{\rm F} (1 - e^{-\delta t}) dt
$$
 (57)

where $\beta = a^{\gamma}_{\alpha}(10^{-6})/\gamma_{\beta}$ by definition and will be denoted as the filter cake resistance index or β index.

Note that for cylindrical septa the face velocity, v, is a function of time and must therefore be expressed in terms of time before Equation 57 can be integrated. This may be accomplished by considering that the surface area of a cylindrical septum is $A_S = 2\pi R_S L_S$, and the gross outer filter area of a cylindrical filter cake of radius R is $A = 2\pi R L_S$. Thus, $A = A_S R/R_S$ and:

$$
v = \frac{Q}{A} = \frac{QR}{A_{S}R} = \frac{qR}{R}
$$
 (58)

However, the outer radius of the filter cake is also a function of time. To derive an expression for R in terms of t, consider that the total volume enclosed within the outer surface area of a filter cake, V_{ϕ} , of radius R is:

$$
V_T = V_S + V_p + V_c = \pi R^2 L_s
$$

where :

$$
V_s
$$
 = volume of septum [L³]
\n V_p = volume of precoat [L³]
\n V_c = volume of filter cake [L³]

$$
V_{\rm s}
$$
 and $V_{\rm p}$ are constants with respect to time, therefore:
\n $dV_{\rm T} = dV_{\rm c} = 2\pi L_{\rm s}R dR$

Equating the above to the right hand side of Equation 56 leads to :

$$
2\pi L_{s}R \, dR = \frac{Q\gamma_{w}C_{F}(10^{-6})}{\gamma_{p}} (1-e^{-\delta t})dt
$$
\n
$$
\gamma_{r}C_{F}(10^{-6}) = -t
$$

and

$$
2R \, dR = \frac{q}{\pi L_s} \frac{\gamma_w C_F (10^{-6})}{\gamma_p} \, (1 - e^{-\delta t}) dt
$$

Remember that $q = Q/A^{}_{S} = Q/2\pi R^{}_{S}L^{}_{S}$. Thus, $Q/\pi L^{}_{S} = 2R^{}_{S}q$ and:
 $2q\gamma^{}_{W}C^{}_{F} (10^{-6})$
 $2R dR = R^{}_{S} \left[\frac{\gamma^{}_{W}C^{}_{F} (10^{-6})}{\gamma^{}_{D}} \right] (1-e^{-\delta t}) dt$ $2\sigma v \sim (10^{-6})$

$$
R \, \mathrm{d}R = R_{\mathrm{S}} \left[\frac{2 \mathrm{d} \gamma_{\mathrm{w}} \mathrm{C}_{\mathrm{F}} \left(10^{-7} \right)}{\gamma_{\mathrm{p}}} \right] \left(1 - e^{-\delta t} \right) \mathrm{d}t
$$

 $2q\gamma_{\rm cr} C_{_{\rm E}}(10^{-6})$ Letting $\phi = \frac{W_F}{W}$ for convenience then:

$$
2R dR = R_{s} \phi (1 - e^{-\delta t}) dt
$$

This differential equation can be integrated as follows:

$$
\int_{R_{\rm O}}^{R} 2R \, \mathrm{d}R = R_{\rm g} \phi \int_{0}^{t} (1 - e^{-\delta t}) \mathrm{d}t
$$

where :

$$
R_0 = \text{outer radius of precoated septum [L]}
$$

= $R_s + L_p$
= $R_s + w/\gamma_p$

$$
[R^2] \bigg|_R^R = R_s \phi [t + \frac{e^{-\delta t}}{\delta}]_0^t
$$

$$
R^2 - R_0^2 = R_s \phi [t + \frac{e^{-\delta t}}{\delta} - 0 - \frac{1}{\delta}]
$$

and

$$
R^{2} = R_{0}^{2} + R_{s} \phi[t - \frac{1 - e^{-\delta t}}{\delta}]
$$

Letting $X = t - \frac{(1 - e^{-\delta t})}{\delta}$, then:

$$
R^{2} = R_{0}^{2} + R_{s} \phi X
$$
(59)

Substituting for R in Equation 58 and likewise for v in Equation 57 leads to:

$$
dH_C = \frac{R_S^2 [q^2 v \beta C_F / g] (1 - e^{-\delta t}) dt}{R_O^2 + R_S \phi X}
$$

and letting

$$
\sigma = q^2 v \beta C_F / g, \text{ then:}
$$

\n
$$
dH_C = \frac{R_S^2 \sigma (1 - e^{-\delta t}) dt}{R_O^2 + R_S \phi X}
$$
\n(60)

X is a function of time, however, if dX is substituted for dt, an equation is derived which can be integrated.

i.e.:
$$
x = t - \frac{1}{\delta} + \frac{e^{-\delta t}}{\delta}
$$

therefore :

$$
dx = dt - 0 - \frac{\delta e^{-\delta t}}{\delta} dt = (1 - e^{-\delta t}) dt
$$

and substituting for $(1-e^{-\delta t})$ dt in Equation 60 gives

 $\bar{1}$

$$
dH_C = \frac{R_S^2 \sigma dx}{R_O^2 + R_S \phi X}
$$

which can be integrated as follows;

$$
\int_{0}^{H_{\rm c}} dH_{\rm c} = \int_{0}^{X} \frac{R_{\rm s}^{2} \sigma dx}{R_{\rm o}^{2} + R_{\rm s} \phi x}
$$

$$
= \frac{R_{\rm s}^{2} \sigma}{R_{\rm s} \phi} \int_{0}^{X} \frac{R_{\rm s} \phi dx}{R_{\rm o}^{2} + R_{\rm s} \phi x}
$$

$$
[H_C]_0^{\text{H}_C} = \frac{R_s \sigma}{\phi} [ln(R_O^2 + R_s \phi X)]_0^X
$$

and

$$
H_C = \frac{R_S \sigma}{\phi} [\ln(R_O^2 + R_S \phi X) - \ln(R_O^2)]
$$

$$
= \frac{R_S \sigma}{\phi} \ln(1 + \frac{R_S \phi X}{R_O^2})
$$
 (6)

The total thickness of precoat and filter cake, L, at time t for cylindrical septa can be determined from Equation 59 and is equal to:

$$
L = R - R_S = \sqrt{R_O^2 + R_S \phi X - R_S}
$$
 (7)

Head loss through filter cake - flat septa

For flat septa, the face velocity v is not a function of time and is equal to q. Therefore Equation 57 can be

written as:

$$
dH_{\rm c} = \sigma (1-e^{-\delta t}) dt
$$

 $= \sigma dx$

and upon integrating:

$$
H_C = \sigma X \tag{8}
$$

Also for flat septa, $dV^c = A^c$ dL^c . Equating this expression to the right hand side of Equation 56 leads to:

$$
A_{\rm s} dL_{\rm c} = \frac{Q\gamma_{\rm w}}{\gamma_{\rm p}} C_{\rm F} (10^{-6}) (1-e^{-\delta t}) dt
$$

$$
= A_{S} \frac{\phi}{2} dX
$$

since :

$$
\phi = \frac{2q\gamma_{w}C_{F}(10^{-6})}{\gamma_{p}} \quad \text{and } \frac{Q}{A_{S}} = q.
$$

Integration leads to:

$$
\int_0^{L_C} dL_c = \frac{\phi}{2} \int_0^X dx
$$

and

$$
L_C = \frac{\phi X}{2}
$$

Therefore :

$$
L = L_p + \frac{\phi X}{2} \tag{9}
$$

APPENDIX C

Working Drawings of the SSCR Filter Apparatus

Note: Unless otherwise noted, all dimensions shown are in inches.

Figure 43. Working drawings of the filter assembly in SSCR apparatus

 $\sim 10^{-11}$

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 $\mathcal{L}^{\text{max}}_{\text{max}}$ and $\mathcal{L}^{\text{max}}_{\text{max}}$

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Figure 44. Working drawings of the gear and value assembly in SSCR apparatus

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 $\sim 10^{-10}$

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FRONT VIEW

SECTION AA

 $\mathcal{L}^{\mathcal{A}}$

Figure 45. Working drawing of the raw water and backwash water holder in SSCR apparatus

 $\mathcal{L}^{\text{max}}_{\text{max}}$ and $\mathcal{L}^{\text{max}}_{\text{max}}$

 $\mathcal{L}^{\text{max}}_{\text{max}}$, where $\mathcal{L}^{\text{max}}_{\text{max}}$

 $\mathcal{L}^{\text{max}}_{\text{max}}$

 \mathcal{A}^{out}

Figure 46. Working drawings of the precoat pot in SSCR apparatus

 $\mathcal{L}(\mathcal{$

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 $\label{eq:2.1} \mathcal{L}(\mathcal{L}(\mathcal{L})) = \math$

 $\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}})$ and $\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}})$. The contribution

the contract of the contract of the contract of

Figure 47. Working drawings of the precoat pot cover in SSCR apparatus

 $\mathcal{L}^{\text{max}}_{\text{max}}$, where $\mathcal{L}^{\text{max}}_{\text{max}}$

 $\mathcal{L}^{\text{max}}_{\text{max}}$, where $\mathcal{L}^{\text{max}}_{\text{max}}$

 $\mathcal{L}(\mathcal{A})$ and $\mathcal{L}(\mathcal{A})$. The set of $\mathcal{L}(\mathcal{A})$

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APPENDIX D

BID Program User Manual

Introduction

The program for Beta Index Determination or the BID Program was prepared to evaluate the filter cake resistance index or β index from the results of a single or series of filter runs. A discussion of the calculation of the β index is made on pages 115-127. For flat septa, the value of B can be determined from the slope of the linear portion of a plot of head loss versus filtration time by the equation:

$$
\beta = \frac{g}{q^2 \nu} \frac{\text{slope}}{C_F} \tag{31}
$$

It is <u>not</u> necessary to know the values of γ_{p} or δ to determine the exact value of β from the results obtained using flat septa.

Using cylindrical septa, the values of β can be determined from the slope of a plot of head loss versus $\ln(1 + R_{\rm g} \phi X/R_{\rm g}^2)$ by the equation:

$$
\beta = \frac{q}{q^2 \nu} \frac{\text{slope}}{C_F} \frac{\phi}{R_S} \tag{61}
$$

It is necessary to know the values of $\gamma_{\rm p}$ and δ to determine the exact value of β from results obtained using cylindrical septa. The effects of using erroneous values of $\gamma_{\rm p}$ and δ on the calculated value of 6 are shown in Tables 7 and 8

(p. 122 and 128, respectively). By estimating the time of inflection from a plot of head loss versus filtration time, Ô may be estimated as :

$$
\delta = 3/t, \tag{32}
$$

The computer program has been written to perform a regression of H_{α} versus t (or X if δ is known) for flat septa, or versus $\ln(1 + R_g \phi X/R_o^2)$ for cylindrical septa, and to determine the value of the filter cake resistance index, β . The program also determines the standard error of estimate, s_r, and the linear correlation coefficient, R, in percent, of the regression equation.

Computer input

To determine the filter cake resistance observed in a filter run, the following data must be read into the computer:

1) Data that remain constant during a filter run are read into the computer in an array named A. This array consists of:

 $A(1) = 1.0$ if the dilution rate, δ , is known; 2.0 if the dilution rate is not known $A(2) = filtration rate, gpm/sq ft$ A(3) = body feed concentration, C_F , mg/l or ppm $A(4)$ = influent water temperature, °C or °F

2) The number of head loss observations made during the filter run is read into the computer under the name NOOBS.

3) The observed head losses (expressed in inches of mercury or feet of water) are read into an array named H, and the corresponding times of filtration (expressed in minutes or hours) are read into an array named T. Thus, the first observed head loss and time of filtration would be identified as $H(1)$ and $T(1)$, respectively.

Flow chart and FORTRAN listing

BID is written in FORTRAN IV computer language (31, 41) for use with the IBM 360/65 computer system at Iowa State University. The flow chart for the program is given in Figure 48 and is followed by the FORTRAN listing.

A detailed explanation of the input statements and output statements is not given since these vary with the computer system used. Basically, an input or output statement consists of a READ or WRITE statement and a FORMAT statement. Some of the symbols used in FORMAT statements

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for the Iowa State University IBM 360/65 computer system are :

' . . . ' All information contained within apostrophes is written on the output sheet in the same form as it is written in the FORMAT statement.

> The first column of each line on the output sheet is reserved for carriage control of the printer. The instruction to leave the first column blank instructs the printer to remain on that line (single space), 0 (i.e., '0') instructs the printer to skip a line (double space), and l (i.e., 'l') instructs the printer to skip to the top of the next output sheet.

- X This instructs the printer to leave a space blank. For example, 7X instructs the printer to skip seven columns or spaces.
- / This instructs the printer to skip to the beginning of the next line. Therefore, //// tells the printer to skip four lines or leave three lines blank.
- F,E,I These are various formats for numerical data. In the FORMAT statement F10.3, F is the type of format, 10 is the size of field, in columns, reserved for the number, and 3 is the number of digits to be written to the right of the decimal point. For example, the number 23.4 would be written under a FlO.3 format as 23.400 with the last digit, 0, in the right column of a ten column field.

Under an E format a number is written in exponential form. The number 23.4 would be written under an ElO.3 format as 0.234E 02 with the last digit, 2, in the right column of a ten column field.

The I format is used for integer numbers. For example, the integer 16 would be written under an 15 format as 16 with the last digit, 6, in the right column of a five column field.

 $\mathcal{A}^{\mathcal{A}}$

A This is a format used for alphameric data. When an alphabetic, numeric, or special

character is read in under an A format, it is given a unique internal cose. For example,
with the IBM 360/65 computer system, the letter
C when read in under an A format is stored as $-1019199424.$

As an example, if the value of a variable named A is 86.45, the output statement:

would result in the following output:

 $\omega^{\mu\nu}$. In

 $\label{eq:2.1} \frac{1}{2} \sum_{i=1}^n \frac{1}{2} \sum_{j=1}^n \frac{$

 $\label{eq:2.1} \frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}^3}\frac{d\mu}{\sqrt{2\pi}}\left(\frac{d\mu}{\mu}\right)^2\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\$

Figure 48. Flow chart for the BID Program

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 $\hat{\mathcal{A}}$

 $\langle \cdot \rangle$

 $\sim 10^{-10}$

In the following explanation of the program, the

numbers to the left refer to the statement numbers shown in

the FORTRAN listing.

- 0001 This statement is necessary to allocate storage for the arrays used in the program.
- 0002 These statements provide for writing the title head-
0003 ing on the output sheet. ing on the output sheet.
- 0004 Statements 4 through 9 instruct the computer to read the first data card and determine if it is a comment
card, i.e., the letter C is in the first column. If card, i.e., the letter C is in the first column. it is a comment card, the information on it is printed out and the next card is read. If it is not a comment card, the computer will continue with Statements 10 and 11 which are instructions for writing
- 0011 headings for the input data.
- 0012 The filter run constants are read into the A array
0015 and printed on the output sheet. and printed on the output sheet.
- 0016 Necessary transformations are performed on the filter run constants. The filtration rate is converted from gpm/sq ft to cu ft/hr/sq ft, the influent temperature is converted from degrees Centigrade to degrees Fahrenheit, the septum radius in feet is calculated, the acceleration of gravity is converted to $ft/hr²$, and the kinematic viscostiy is calculated in sq ft/hr.

The equation used to compute the kinematic viscosity of water from the temperature (°F) was obtained by fitting a portion of an ellipse to tabulated values in a handbook (21). The values of viscosity obtained using the equation have been compared with handbook values and found to be acceptable within the range of temperature used in filtration.

If the influent temperature is input in degrees Fahrenheit, Statement 18 should be omitted, i.e., the 0021 card should be removed from the deck.

0022 The number of observations, NOOBS, is read into the computer. Then the head loss and time data are read 0025 until NOOBS number of observations have been read.

- 0026 Each head loss value is converted from inches of mercury to feet of water, and each value of time is converted from minutes to hours. If these conversions are unnecessary. Statements 30 and 31 should be eliminated, i.e., the cards should be removed from 0031 the deck.
- 0032 If the dilution rate is known, the value of X is calculated for each value of time. If the filter septa are flat, $R_S=0$, the calculated values of X are stored in an array named B. If the filter septa are cylindrical, the term $\ln{(1 + R_s \phi X/R_0^2)}$ is calculated for each value of X and stored in array B.
- If the dilution rate is not known, values of time t, 0039 are used in place of X.
- 0040 The head loss values in feet and the times of observation in hours are printed out along with the values of X (or t) for flat septa or the natural log term 0049 for cylindrical septa.
- 0050 A simple linear regression is performed on the plot of X or $\ln(1 + R_g \phi X/R_0^2)$ versus head loss.

The regression formula for a simple linear relationship is (53) :

 $\hat{Y} = b_0 + b_1 X$

where $b_1 = \frac{\sum xy}{\sum x_i^2}$

and $b_0 = \overline{Y} - b_1\overline{X}$

where $x = X - \overline{X}$

and $y = Y - \overline{Y}$.

Notation: X , $Y =$ observed values

 \overline{X} , \overline{Y} = average of observed values

 \bar{Y} = computed or estimated values of Y

The notation used in the program is: $B_T = X =$ values of X (or t) for flat septa or natural log terms for cylindrical septa $H_T = Y = head$ loss values $SX = \Sigma X$ $SY = \Sigma Y$ $XBAR = \overline{X}$ YBAR = \overline{Y} $X_T = x = X - XBAR$ $Y_T = Y = Y - YBAR$ $XSO = \Sigma x^2$ $YSQ = \Sigma y^2$ $XY = \Sigma xy$ $SLOPE = b_1$

0071 HP = b_0 = head loss through the precoat

- 0072 The S index is calculated, in Statement 73 for flat septa or Statement 77 for cylindrical septa from the slope of the regression line. The calculated values of β , ϕ , R_s and R_o are printed out along with the various 0081 headings
- 00 82 Head losses are computed using the regression equation. The actual (DIFF) and percent (P) differences between the observed and computed head losses are also com-0090 puted and printed out.
- 0091 The standard error of estimate (SEE) is calculated from the formula (53):

$$
s_E = \sqrt{\frac{\sum (Y - \hat{Y})^2}{n - 2}}
$$

where Y - \hat{Y} is the actual difference between observed and computed values of head loss. The value of $(Y - \hat{Y})^2$ was determined in Statement 87.

n = number of observations, NOOBS.

0092 The linear correlation coefficient, R, in percent, is calculated from the formula (53):

$$
R = \frac{xy}{\sqrt{\left(\Sigma x^2\right) \left(\Sigma y^2\right)}} (100)
$$

where x and y are as defined previously.

0093 The standard error of estimate and linear correlation
0094 coefficient are printed out. coefficient are printed out.

0095 The computer reads the card following the head loss and time data cards and determines if there is an asterisk (*) in the first column. If there is, control is transferred back to Statement 4 and the next set of filter run data is processed. If there is not an asterisk in the first column, the program is stopped.

Input format and examples

Any number of comment cards, indicated by a C in the first column, may be inserted before each set of filter run data (Refer to the example input on pages 298 and 299). The information on each comment card is printed on the output sheet. After the last comment card, a card with DATA written in columns 1-4 should be inserted. If no comment cards are used, this card should still be present.

The next card contains the filter run constants $(A(1),$ $A(2)$, ..., $A(8)$ in eight 10-column fields. The decimal point must be given for each constant; however, the constant may be located anywhere within the proper field.

The following card contains the number of observations.

296

This number must be written in integer form with the last digit in Column 5. This card is followed by a series of cards containing the observed head losses and filtration times. On each card, the observed head loss must be located within Columns 1-10, and the filtration time must be located within Columns 11-20. The decimal point must be given for each head loss and time.

The computer will analyze the results of any number of runs in sequence. To accomplish this, a card with asterisks in Columns 1-5 must be inserted between each set of filter run data. The last set of data should be followed by a card with END written in Columns 1-3.

The following examples give the input and output for two sets of filter run data. The 8 indices are also calculated manually for comparison with the computer results.

80 COLUMN DATA SHEET

 $\mathcal{L}^{\text{max}}_{\text{max}}$ and $\mathcal{L}^{\text{max}}_{\text{max}}$

 $\ddot{}$

 \sim \sim

 $\mathcal{A}^{\mathcal{A}}$

 $\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}})$ and the contract of the contrac

133HS V1VO NR0103 08

 $\frac{1}{2}$

299

 $\mathcal{L}^{\mathcal{L}}$

DETERMINATION OF BETA INDEX BRIDGES - 1967

EXAMPLE NUMBER 1 BRIDGES AND ARORA RUN NUMBER 53 DISTILLED WATER PLUS 33.9 MG/L UNSETTLED BALL CLAY HYFLO SUPEP-CEL FILTER AID SSCR FILTER (FL&T SEPTUM)

INPUT DATA

 ~ 100 km s $^{-1}$

 $\sim 10^{11}$

 $\sim 10^{-1}$

 $\sim 10^{11}$

STANDARD ERROR OF ESTIMATE

 \sim \sim

0.C24

 $\omega_{\rm{eff}}$, and

R = LINEAR CORRELATION COEFFICIENT 100{R) = 99.873

 α and α and α

EXAMPLE NUMBER 2 BRIDGES (1966) FILTER RUN NUMBER 30 COAGULATED, FLOCCULATED, AND SETTLED LAKE WATER AT CRESTON, IOWA INFLUENT TURBIDITY = 8.2 JTU HYFLO SUPER-CEL FILTER AID MOBILE TREATMENT UNIT (CYLINDRICAL SEPTA)

INPUT DATA

DATA PRIOR TO REGRESSION

 \sim

 $\mathcal{L}(\mathcal{A})$ and $\mathcal{A}(\mathcal{A})$

STANDARD ERROR OF ESTIMATE

 \sim

 0.357

 $R = LINEAR CORRELATION CCEPTICIENT$ 100(R) = 99.892

ü.

Example Number 1:

Bridges and Arora Run Number 53 (Appendix A, Table 32)

Distilled water plus 33.9 mg/l unsettled Ball clay Hyflo Super-Cel filter aid

SSCR filter (flat septum)

Data: Filtration rate, q = 1.05 gpm/sq ft

Body feed rate, $C_p = 80$ mg/1

Influent temperature = 26.1 °C

Calculations :

The slope of the head loss versus time curve for this filter run (Figure 49) becomes constant at a value of 0.0625 cm Hg/min after 24 minutes of filtration. Now

$$
\beta = \frac{g}{q^2 \nu} \frac{\text{Slope}}{C_F} \tag{31}
$$

where

 $q = 1.05$ gpm/sq ft = $1.05 \frac{gal}{min\ sq\ ft} \times 60 \frac{min}{hr} \times \frac{cu\ ft}{7.48\ gal}$ $= 1.05 (8.02)$ $= 8.421 \text{ ft/hr}$ $g = 32.2 \text{ ft/sec}^2$ $= 417.3 \times 10^6 \text{ ft/hr}^2$

and

$$
\nu = \frac{\text{viscosity}}{\text{density}} = \frac{\mu}{\gamma_w}
$$

where

$$
\mu = 8.718 \times 10^{-3} \text{ poise From handbook (21)}
$$

= (8.718 × 10⁻³)242 lb/hr ft
= 2.110 lb/hr ft

 $\gamma_w = 62.23$ lb/cu ft From handbook (21)

therefore,

$$
v = \frac{2.110}{62.23} = 3.391 \times 10^{-2}
$$
 sq ft/hr

Note: The value of v calculated from the equation used

Figure 49. Head loss versus time curve for Example 1

 $\mathcal{L}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}$

 $\mathcal{L}^{\text{max}}_{\text{max}}$ and $\mathcal{L}^{\text{max}}_{\text{max}}$

 ~ 100 km s $^{-1}$

 $\mathcal{L}^{\text{max}}_{\text{max}}$ and $\mathcal{L}^{\text{max}}_{\text{max}}$

 $\mathcal{A}^{\mathcal{A}}$

and

in the computer program is 3.393 x
$$
10^{-2}
$$
 sq ft/hr.
and
Slope = 0.0625 $\frac{\text{cm Hg}}{\text{min}} \times \frac{1.05 \text{ ft water/in. Hg}}{2.54 \text{ cm/in.}} \times 60 \frac{\text{min}}{\text{hr}}$

 $= 1.55$ ft water/hr

therefore

$$
\beta = \frac{417.3 \times 10^{6}}{(8.421)^{2} (3.391 \times 10^{-2})} \frac{1.55}{80}
$$

$$
= \frac{338 \times 10^{4} \text{ ft}^{-2}}{}
$$

The value using the computer is 338.5×10^4 ft⁻² with a linear correlation coefficient of 0.99873 and standard error of estimate of 0.024 ft.

Example number 2:

Bridges (1966) Filter Run Number 30 (Appendix A, Table 27) Coagulated, flocculated, and settled lake water

at Creston, Iowa. Influent turbidity = 8.2 JTU.

Hyflo Super-Cel filter aid

Mobile treatment unit's filter (cylindrical septa)

Data: Septum diameter = 3.5 in

Precoat weight, $w = 0.20$ lb/sq ft Precoat density, $\gamma_p = 15$ lb/cu ft Filtration rate, $q = 0.98$ gpm/sq ft Body feed rate, $C_F = 28.42$ mg/l Influent temperature = 26.1 °C Initial dilution rate, $\delta = 2/hr$

Calculations :

The diameter of each filter septum = 3.5 in. Therefore,

$$
R_{\rm s} = \frac{3.5 \text{ in.}}{2(12) \text{ in.}/\text{ft}} = \frac{0.146 \text{ ft}}{}
$$

$$
R_o = R_s + L_p
$$

where

 $\bar{\mathcal{A}}$

$$
L_p = w / \gamma_p
$$

Therefore,

$$
L_p = \frac{0.20 \text{ lb/sq ft}}{15.0 \text{ lb/cu ft}}
$$

$$
= 0.013 \text{ ft}
$$

and

 $\hat{\mathcal{L}}$

R_O = 0.146 + 0.013
=
$$
\frac{0.159 \text{ ft}}{2 \text{ q} \gamma_{\text{w}} \text{ C}_{\text{F}} (10^{-6}) / \gamma_{\text{p}}}
$$

where

 $q = 0.98$ gpm/sq ft $= 0.98$ gal \times 60 min \times cu ft min sq ft x hr x 7.48 gal $= 0.98 (8.02)$ $= 7.861 \text{ ft/hr}$ $\gamma_p = 62.4$ lb/cu ft $C_F = 28.42$ mg/1 or ppm $\gamma_n = 15.0$ lb/cu ft P Therefore,

$$
\phi = \frac{2(7.861) (62.4) (28.42) (10^{-6})}{15.0}
$$

$$
= \frac{1.858 \times 10^{-3} \text{ ft/hr}}{}
$$

and,

 ~ 10

$$
\ln(1 + R_s \phi X/R_0^2) = \ln(1 + \frac{(0.146)(1.858 \times 10^{-3})}{(0.159)^2} x)
$$

$$
= \ln(1 + (1.0698 \times 10^{-2}) \text{ X})
$$

For manual calculations, X may be approximated by t. The value of the above quantity is shown below for each value of t.

 $\hat{\boldsymbol{\beta}}$

 $\texttt{H}_{\texttt{C}}$ is then plotted versus ln(1 + $\texttt{R}_{\texttt{S}}\phi\texttt{t}/\texttt{R}_{\texttt{O}}^{2}$) as in Figure 50. The slope of the estimated regression line is determined to be 460 ft. The slope is equal to R^c_S 0/ ϕ . Therefore

$$
\sigma = \frac{460 \phi}{R_s}
$$

= $\frac{460 (1.858 \times 10^{-3})}{0.146}$
= $\frac{5.854 \text{ ft/hr}}{0.146}$

$$
\sigma = q^2 v \beta C_F / g
$$

Therefore,

$$
\beta = \frac{q^2 v C_F}{q^2}
$$

where

$$
q = 7.861 \text{ ft/hr}
$$

 $g = 32.2 \text{ ft/sec}^2 = 417.3 \times 10^6 \text{ ft/hr}^2$

Figure 50. Head loss versus the natural log term

 \sim

 $\mathcal{L}^{\text{max}}_{\text{max}}$

 $\sim 10^7$

 \sim

 \sim

 \sim

 $\sim 10^6$

and since the temperature is the same as in Example 1,

$$
v = 3.391 \times 10^{-2}
$$
 sq ft/hr

and,

$$
\beta = \frac{(5.854) (417.3 \times 10^6)}{(7.861)^2 (3.391 \times 10^{-2}) (28.42)}
$$

$$
= \frac{41.02 \times 10^6 \text{ ft}^{-2}}{}
$$

The value obtained using the computer with $\delta = 2/hr$ is 43.960 ft^{-2} with a linear correlation coefficient of 0.99892 and standard error of estimate of 0.357 ft.

APPENDIX E

MAIDS Program User Manual

Introduction

The program for Manipulation and Interpretation of Data Systems or the MAIDS Program was designed to perform a multiple regression of from two to eight variables to fit the linear model,

 $T1 = B1 + B2*T2 + B3*T3 + ... + Bn*Tn$

where T1 is the dependent variable, T2 ... Tn are independent variables, B1 ... Bn are regression coefficients. n can be from two to eight.

If the model (the equation of the curve being fitted) is not linear, some transformation or combination of transformations must be performed to make the model linear. For example, the model

 $T1 = 10^{B1} * T2^{B2} * T3^{B3}$

can be made linear by taking the logarithm (base 10) of each variable.

 $log T1 = B1 + B2*log T2 + B3*log T3$

This type of transformation is necessary for determining β prediction equations.

Any desired transformation is possible with MAIDS. Thus, the program is very useful for reducing and printing out tabular data, even though a regression analysis is not desired.

Input card forms

Data used by MAIDS is read into the computer on 80 column punched cards. The card formats used with MAIDS are listed in Table 33.

1) KAN array card:

The KAN array card is always the first input card read by the computer. Each letter, symbol, and digit on the card is stored in an array named KAN. Therefore, KAN(1) = C , KAN(2) = L, KAN(3) = D, ..., KAN(50) = U. The KAN array is then used by the computer to read the remaining input cards. This is done by comparing each letter, symbol, or digit on the input card with each element of the KAN array. For example, if the letter in the first column of an input card matches KAN(l), i.e., the letter C is punched in Column 1 of the input card, the card is a comment card. Therefore, the computer will print out the information contained on the card and then read the next input card.

2) Comment and label cards

Cards with C or L punched in Column 1 are ignored by MAIDS; however, the information on the cards is printed on the output sheet. These cards can be used to make comments or to label

Table 33. MAIDS input card formats

 $\mathcal{L}_{\mathcal{A}}$

 ~ 100

 $\sim 10^7$

319

 \sim \sim

 \sim

values in tables. Any characters can be punched in Columns 2 to 80 of a comment or label card and any number of comment cards can be used.

3) Data cards

 $\Delta \sim 10^{11}$, and $\Delta \sim 10^{11}$

A card with Column 1 left blank is used to input observed values of the variables (Tl, T2, T3, .../ Tn) used in the regression analysis. On an 80 column data card there are eight 10-column fields available for up to eight observed variables. The first field actually contains only nine columns since the first column must be left blank. Each observed variable may be punched anywhere within a particular 10-column field. The decimal point must occupy one column. A plus sign may or may not be punched before a positive valued variable. Each data card contains one observation of each variable being read in. A particular variable must be punched in the same 10-column field on all data cards. If a 10-column field on a particular data card is blank, that field is ignored; a blank is not interpreted as zero.

A card with A punched in Column 1 may be used to read from one to eight different constants into the computer. Each constant must be punched anywhere within one of the eight 10-column fields with the decimal point occupying one column. These constants are used in transforming the observed variables.

4) Transformation card:

A card with T punched in Column 1 is a transformation card. The letter t shown in Table 33 represents the transformation symbols given in Table 35.

The observed variables input on a data card are denoted witin the program as Xk where $k = 1, 2, 3, ...$, 8. When a transformation is applied to Xk, the transformed variable is denoted as Tk. Tk can be transformed to a new variable which is also denoted as Tk.

The variable to be transformed by a particular transformation as specified on a transformation card is determined by in which of the eight 10-column fields the transformation symbol occurs. The symbol may be punched anywhere within the proper 10-column field. For example, in Table 34 four observations of three variables X1, X2, and X3, are read in. The

Ť.

Index no. ^a	Symbol	Meaning
\mathbf{I}	LN	natural log
	LOG	base 10 log
	SIN	sin
$\begin{array}{c} 2 \\ 3 \\ 4 \end{array}$	\cos	cos
	EXP	e.g. EXP punched in columns $14-16$ means e^{T2}
$\frac{5}{6}$	$+Ti$	add i-th transformed variable
$\overline{7}$	$+Xi$	add i-th original (read in) variable
8	$+p$	add p (p = constant or Ai where Ai = i-th A value)
9	$-Ti$	
10	-xi	subtract
11	$-p$	
12	*Ti	
13	*xi	multiply
14	4^{\star}	
15	ABS	absolute value
16	/Ti	
17	/Xi	Divide
18	/p	
19	$=Ti$	
20	$=Xi$	set equal to
21	$=$ p	
22	5T1	
23	\$Xi	raise to power
24	\$p	
25	CUM	each element = sum of itself and all before
26	DELTA	each element = increment from previous element
27	MEAN	each element is mean of this element and pre- ceding element
28	ORDER	e.g., if punched in Columns 22-26, rearranges
		transformed variables and original variables
		such that the new T3 would be in ascending
		order.
29	JANET	
30	LEANN	variable subroutines written for special
31	SUZIE J	transformations not included in 28 above transformations
		$Xi =$ ith column of data read in
		$Ti = ith$ column of the current transformed variable.
		$p =$ either a signed constant
	or Ai	
	$Ai = ith A value.$	

Table 35. List of transformations allowed in MAIDS

a
See page 337.

first transformation card instructs the computer to divide XI by 10.0, add 5.0 to X2, and subtract 60.0 from X3. This results in the transformed variables:

The second transformation card instructs the computer to subtract T3 from T2. This gives the transformed variables :

Transformations JANET, LEANN, and SUZIE are for special transformations. These are variable subroutines which can be written by users familiar with the FORTRAN computer language to perform a transformation which is not included in Table 35. These variable subroutines allow the user to perform any desired transformation on an original variable, Xk, or previously transformed variable, Tk. Up to three special transformations can be made, one for each of the

three variable subroutines.

Most FORTRAN systems contain standard library functions, such as arc since, hyperbolic sine, etc., which have not been included in the list of MAIDS transformations. These may be used in a variable subroutine. For example, suppose it is desired to transform the variable T2 as follows:

 $T2 = T2 * arctan (A3/X7)$.

This can be done by punching JANET in the second 10 column field of a transformation card (T in Column 1) and including the following subroutine with the MAIDS program subroutines.

```
SUBROUTINE JANET (J) 
   COMMON B(10,10),X(8,400),T(8,409),NM(80),JT(100), 
  IIT(IOO) ,PT(100) ,IXT(100) ,A(8) ,MN(75) ,NOOBS, 
  2N0VAR, LEE, KFLAG, KAN(50) 
   DO 1 I = 1, NOOBS
1 \quad T(J, I) = T(J, I) * \text{ATAN}(A(3)/X(7, I))RETURN 
   END
```
The SUBROUTINE and COMMON statements in the example above are similar for any variable subroutine (JANET, LEANN, SUZIE), except the name of the subroutine changes. The value of the variable J (two in this example) is the number of the 10 column field that JANET was punched in on the transformation card. Note that JANET was punched in the second 10-column field since the second variable, T2, is to be transformed. The first subscript of T (or X) is the variable number (1 to 8)

and the second is the element number or observation number, i.e., T (2, 10) would be the tenth observation of T2. A(3) is the number read in from the third 10-column field of the A card. In the COMMON statement of the subroutines, B, NM, JT, IT, PT, IXT, MN, and LEE are arrays used in other parts of MAIDS and are included here as dummy variables to align COMMON sotrage. NOOBS is the number of observations of variables read in and NOVAR is the number of variables, ranging from one to eight. KFLAG is an error indicator. Its usual value is one but it is assigned a value of two if an error is encountered. Therefore, if KFLAG has a value of two when control is returned from a subroutine to the main program, an error statement is printed as follows: **UNIDENTIFIABLE CHARACTER **. KAN is the KAN array from the first input card and ATAN is the FORTRAN library function for computing arc tangent.

Whenever a variable subroutine is used, the corresponding previous variable subroutine having the same name must be removed from the program. This is necessary because there cannot be two subroutines with the same name in the same program. For some FORTRAN compilers it is required that every subroutine which is referenced in the program be included. Therefore, even if a variable subroutine is not used it should nevertheless be included in the program as a dummy subroutine that actually does nothing. For example:

SUBROUTINE JANET (J) COMMON B(10,10), X(8,400), T(8,409), NM(80), JT(IOO), IIT(IOO), PT(IOO), IXT(IOO), A(8), MN(75), NOOBS, 2N0VAR, LEE, KFLAG, KAN(50) DUMMY = 1 RETURN END

5) V* card:

A card with V* punched in Columns 1 and 2 respectively and a number, i (i=1 to 8), punched in anywhere within Columns 3 and 10 specifies the number of variables to use when a PRINT, REGRESSION, or PACOCO statement is executed. If not specified, the number of variables is taken as eight.

6) PRINT card:

A card with PRINT punched in Columns 1 to 5 instructs the computer to print out the values of the first i transformed variables (Tl, T2, T3, ..., Ti). The value of i is specified on the V* card.

7) REGRESSION card:

A card with REGRESSION punched in Columns 1 to 10 instructs the computer to determine the regression coefficients of the linear model,

 $T1 = B1 + B2 * T2 + B3 * T3 + ... + Bi * Ti.$

The method of least squares is used to determine the regression coefficients, Bl, B2, B3, ..., Bi. Included in the

output are the transformed variables and the partial correlation coefficients. The computed (by the regression equation) and the observed values of the dependent variable (Tl) are printed along with the standard error of estimate and multiple correlation coefficient.

8) PACOCO card:

A card with PACOCO punched in Columns 1 to 6 instructs the computer to determine and print the partial correlation coefficients for the first i (as specified on a V* card) transformed variables. As noted above, the partial correlation coefficients are printed when a REGRESSION card is executed; however, a regression is not performed when a PACOCO card is executed. Regression coefficients, Bl, B2, ..., Bn, are not determined.

9) ***** card and *DATA card:

A card with ***** punched in the first five columns instructs the computer to perform the specified operations on the group of data read in prior to the ***** card. The same analysis can be performed on any number of data groups. Each data group must be followed by a ***** card. The specified operations are contained on the operation cards (T, V*, PRINT, REGRESSION, and PACOCO cards). The operation cards are included with the first group of data and need not be repeated for additional data groups on which the same

operations are to be performed.

A *DATA card is used when it is desired to change only the values of constants read in via a card with A punched in the first column. An A card containing a new set of constants should follow the *DATA card. This instructs the computer to perform the specified operations on the original set of data using the new constants. An example problem using the *DATA card will be given later.

10) END card:

END punched in the first three columns of a card instructs the computer to clear its memory of operation statements. This card is used to separate the series of data groups on which different operations are to be performed. An END card is followed by another set of operation cards, (T, V*, PRINT, REGRESSION, and PACOCO) and corresponding data groups.

11) STOP card:

Whenever a card with STOP punched in the first four columns is encountered, the computer run is terminated. This should be the last card to be read by the computer.

FORTRAN listing

MAIDS is written in FORTRAN IV computer language (31, 41) for use with the IBM 360/65 computer system at Iowa

State University. The FORTRAN listing of the program is given in this section. Some of the symbols used in FORMAT statements for the Iowa State University IBM 360/65 computer system have been explained in Appendix D.

A complete explanation of the FORTRAN listing for MAIDS would be very lengthy. Therefore, only a brief explanation of the basic purposes of the main program and each subroutine is given. Figure 51 is a schematic diagram showing the relationships between the various subroutines of MAIDS. The arrows in Figure 51 point to the subroutine which is called. Input subroutines are herein defined as subroutines which are used for reading and interpreting input cards. Operation subroutines are defined as subroutines used to perform the specified operations determined by the input subroutines.

Figure 51. Schematic diagram of MAIDS subroutines

 $\sim 10^{-1}$

 $\mathcal{A}^{\mathcal{A}}$

 \mathcal{A}

MAIN program

The first card in the data deck is read and the information on it is stored in the KAN array. The succeeding card is then read and the character in the first column is determined by comparison to the KAN array. Control is then transferred to an appropriate subroutine or statement number. For example, if the first character on the card is T, (TRANSFORMATION card), control is transferred to subroutine ALICE which determines what transformation is to be performed. The first character on each input card must correspond to one of the input card forms, i.e., the first character must be either C, L, blank, A, T, P, R, V, *, E, or S. If any other character is punched in the first column of a card, the error statement "UNIDENTIFIABLE CHARACTER" is printed. If an unidentifiable character is read, the computer will continue reading cards until either an END or a STOP card is read.

G MAIN PROGRAM — MAIDS COMMON B(10,1C),X(8,400),T(8,409),NM(80),JT(100), 1IT{100),PT(100),IXT(100),A(8),MN(75),NOOBS,NOVAR,LEE, 2KFLAG,KAN(50),LAY,FIELD(4,6),XA(4,5),YA(4,5),LA(4,5), 3M&(4,5) REAO(ltlOO) (KAN(K),K=1,50) 100 FORMAT(ROAl) 1 l,EE=0 LAY=0 nn 2 1=1,8 **? All)=0.0 N0VAR=8 NGnBS=0 KFLAG=1**

```
WRITE(3,200)
200 FORMAT("1",//////," M A I D S ",/," DILLINGHAM 1964 --*,
   1'RESETT 1966--BRIDGES 1967', ///)
  3 READ(1,100) (NM(1), I=1,80)WRITE(3,300) (NM(I),I=1,80)
300 FORMAT ('0',80A1)
    004K=1,14IF(NM(1)-KAN(K)) 4,5,4
  4 CONTINUE
    GG TO 22
  5 GO TO (3,3,6,7,8,6,16,17,27,1,18,20,21,6), K
  6 CALL DORIS
    GO TO (3,24), KFLAG
  7 CALL ALICF
    GO TO (3,24), KFLAG
  8 LEE=LEE+1
    IF(NM(2)-KAN(6)) 10,9,10
  9 IXT(LEE) = -6GN TO 3
 10 IF(NM(2)-KAN(8)) 12, 11, 12
 11 IXT(LEE) =-5
    GO TO 312 IF(NM(2)-KAN(50)) 24, 13, 24
 13 IXT(LFE) = -414 DO 15 KK=1,75
 15 MN(KK)=NM(KK+5)
    GO TO 3
 16 CALL MARIE
    GO TO (3,24), KFLAG
 17 LEE=LEE+1
    IXT(LEE)=0GC TO 3
 18 GO TO 24
 20 GO TO 24
 21 GO TO 24
 22 IF(NM(1)-KAN(31)) 24,23,24
 23 CALL ANN
    GO TO 3
24 WRITE(3,400) (NM(I), I=1,80)
400 FORMAT('0',5X,'**UNIDENTIFIABLE CHARACTER**',80A1)
25 READ(1,100) K
    IF(K-KAN(10)) 26, 1, 26
26 IF(K-KAN(°)) 25,27,25
27 STOP
    FND
```
Subroutine FRAN

Subroutine FRAN is used by subroutines DORIS, MARIE, and ALICE to read numerical data. When a card is read by the MAIN program, each character is stored in the array NM. For example, suppose 1.24 is punched in Columns 14 to 17 of a data card. This would not be read in as the number 1.24, but as individual characters: $NM(14) = 1$, $NM(15) = .$, $NM(16) = 2$, and $NM(17) = 4$. Subroutine FRAN is used to combine these individual characters into one numerical value. The KAN array is used for comparison to determine the numerical value of individual characters. The argument P is the number that is determined, N is the number of the column in which either a plus sign, a minus sign, or the first digit of the number if punched in, and L is the number of the last column in the 10-column field containing the number.

MAIDS could have been written to read numerical values directly. However, the method used allows each input card to be read under the same format. This increases the simplicity of the input card forms and the versatility of the program.

```
SUBPOUTINE FRAN(P,N,L)
  CCMMOM P(10,10),X(9,400),T(8,409),NM(80),JT(100), 
 1TTflOOy t PT(IOO)tlXT(lOO)t A(B)t MN(75),NOOBS,NOVAR,LEE, 
 ?KFLAG,KAN(50) 
  SN=1 .0 
  NUMBR=0 
  KPT=-20 
  IF(M-L) 1,1,3 
3 KFLAG=2 
  RETURN
```
1 on **10** T**=N**,L **J=NM{Î) DO 2 K=14,27 IF(J-KAN(K)) 2,4,2 2 CONTINUE GO TO 3 4 K=K-18 IF(K! 5,9,9 5 K=-K GO TO (6,8,10,10),K 6 IF(KPT) 7,3,3 7 KPT=0 GO TO 10 S SN=-i.n GO TO 10 9 NUMBB=NUMRR*10+K KPT=KPT+1 10 CHNTTNUF IF(KPTÎ 11,11,12** 11 n=i.o **GO TO 13 12 D=FLOAT(10**KPT) 13 P=SN*FLPAT(NUMBR)/D RETURN END**

Subroutine DORIS

Control is transferred to subroutine DORIS whenever a data card (blank or A in first column) is read by the MAIN program. If the first column is blank, the number of observations of variables (NOOBS) is increased by one. Each successive column of the data card is checked until a column that is not blank is found. The number of this column, (K), is used to calculate the number of the 10-column field (J) and the number of the last column in the field (L) in which Column K is located. Subroutine FRAN is then used to

determine the numerical value of the observed variable X(J,NOOBS), or constant, A(J). This process continues until the data in each field $(J = 1, 2, 3, ... , 8)$ have been determined. Note that a blank field is not interpreted as zero but is left blank. Also, the values of observed variables are stored in both of the two dimensional arrays, X(J,NOOBS) and T(J,NOOBS). When a transformation is applied to an original variable, Xk (k= $1, 2, 3, \ldots$, 8), or a transformed variable, Tk, the new transformed value is stored in the T array. Thus, the values of Xk stored in the X array remain equal to the original variables read in.

```
SUBROUTINE OORÎS 
  COMMON B(10,10),X(8,400),T(8,409),NM(803,JT(100), 
 IIT(100), PT(10C), IXT(100), A(8), MN(75), NOOBS, NOVAR, LEE,
 2KFLAG,KAN(5Q) 
  L=2 
  1F(NM(1)-KAN(6)) 1,2,1 
1 NOnBS=NOOBS+l 
2 IF(L-80) 3,3,5 
3 DO 4 K=L,80 
  TF(NM(K)-KAN(14)) 6,4,6 
4 CONTTNUF 
5 PFTUPN 
6 J={K+9)/10 
  L=J*10 
  CALL FRAN (P,K,L) 
  L=L + 1 
  1F(NM(1)-KAN(6)) 8,7,8 
7 A(J)=P 
  GO TO 2 
8 T(J,NOCBS)=P 
  X(J,NnOBS)=P 
  GC TO 2 
  F NO
```
Subroutine MARIE

Control is transferred to subroutine MARIE from the MAIN program whenever a V* card is read. Subroutine FRAN is then called to determine the numerical value of the number of variables, NOVAR, punched between Columns 3 and 10 of the V* card. The IXT array and variable subscript, LEE, are used to keep track of the sequence of operations to be performed on the data for each job.

SUBROUTINE MARIE COMMON 8(10,10)»X(8,^00),T(8,409),NM(80»,JT(IOO) , $117(100)$, $PT(100)$, $IXT(100)$, $A(8)$, $MN(75)$, NOOBS, NOVAR, LEE, 2KFLAG,KAN(50) CALL FRAN (D,3,10) **NGVAR=P LEE=LFF+1 IXT(LFF)=-1 PT(LEE**)=NCVAR $IF(NOVAR-8)$ 2,2,1 1 KFLAG=2 2 RETURN END

Subroutine ALICE

Subroutine ALICE is called by the MAIN program whenever a TRANSFORMATION card is read. This subroutine uses the KAN array to determine which transformation is to be applied and sets the variable, INDEX, equal to the index number corresponding to that transformation, (see Table 35). This index number is used later by subroutine GWEN which performs the actual

transformation. For transformations six through 24, subroutine FRAN is used to determine the numberical values (i or p) in the transformation.

```
SLBPOUTINF ALICE 
   COMMON P(lOtlC) ,X(8,400) »T(8,409),NM(80 J »JT(100)» 
  1IT(100)tPT(100),IXT(100),A(8),MN(75),NOOBS,NOVAR,LEE, 
  2KFLAG,KAN(50) 
   P=0 
   M=2 
 1 IF(M-RO) 2,2,4 
 2 no 3 MM=M,R0 
   1F(NM(MM)-KAN(28)} 5,3,5 
 3 CONTINUE 
 4 RETURN 
 5 J={MV+o)/10 
   M=J*10 
   I=NM(MM)L=NM(MM+1) 
   ISN=1 
   DO 6 K=29,43 
   IF(I-KAN(K)) 6,8,6 
 6 CONTINUE 
 7 KFLAG=2 
   RETURN 
 8 K=K-2P 
   GO TO (9,10,11,12,13,14,21,27,31,35,36,37,38,39,40),K 
 9 IN0EX=5 
   GO TO 15 
10 INDEX=8 
   GO TO 15 
11 INDEX=11 
   GO TP 15 
12 INDEX=15 
   GO TO 
13 INDEX=18 
   GO TO 15 
14 INDEX=21 
15 DO 16 K=45,46 
   INDEX=IN0EX+1 
   TF(L-KAN(K)Î 16,20,16 
16 CONTINUE 
   TNDEX=INDEX+1 
   IF(L-KAN(39)) 17,19,17 
17 L=MM+1
```
TSN=0 18 CALL FRAN (P,L,M) GO TO (61,7),KFLAG 19 ISN=-1 **20 L=MM+2 GO TO in 21 TF(L-KAN(A2)) 22,24,22 22 IF(L-KAN(38)) 23,25,23 23 T«=(L-KAN(48)) 7,26,7 24 INDEX-2 GO TO 41 25 INDEX=30 GO TO 41 26 TNDEX=1 GO TO 41 27 IF(L-KAN(49)) 28,29,28** 28 TF(L-KAN(50)) 7,30,7 **29 TNDEX=3 GO TO 41 30 TN0EX=31 GO TO 41 31 TF(L-KAN(42)) 32,33,32 32 IF(L-KAM(50)) 7,34,7 33 TNDEX=4 GO TO 41 34 INDEX=25 GO TO 41 35 TNDFX=5 GC TO 41 36 INDEX=15 GO TO 41 37 TNDEX=26 GO TO 41 38 INDEX=27 GO TO 41 39 IN0FX=2P GO TO 41 40 INDEX=2° 41 LFE=LEE+1 JT(LrE)=J PT(LEE)=P IT(LEFÎ=ÎSN*IFIX(P) IF(ÎABS(ÎT(LEE))-8) 43,43,42 42 TF(L-MM-l) 43,43,7 43 TXT(LEE)=TNDEX M=M+1 GO TO 1 F NO**

Subroutine ANN

Subroutine ANN is called by the MAIN program whenever a ***** card or *DATA card is read. This subroutine then instructs the other operation subroutines to perform the desired sequence of operations.

```
SUBROUTINE ANN 
    CCMMON B(10,10),X(8,400),T(8,409),NM(80),JT(100), 
   1IT(100),PT(100),IXT(100),A{8),MN(75),NOOBS,NOVAR,LEE, 
   2KFLAG,KAN(50),LAY 
    LAY=0 
    DO 13 K=1,LEE 
    IF(ÎXT(KÏ) 2,1,9 
  1 CALL RENE 
    GO TO 13 
  2 L=-IXT(K) 
    GO TO (3,4,5,6,7,8),L 
  3 N0VAR=PT(K) 
  4 GO TO 13 
  5 NOOBS = PT(K) 
  6 GO TO 13 
  7 CALL KATHY 
    GO TO 13 
  8 CALL NORMA 
    GO TO 13 
  9 J=JT(K) 
    I=IT(K) 
    P=PT(K) 
    IX=IXT(K) 
    IF(I) 10,12,12 
 10 I=-I 
    P=A(I) 
 11 I=-T 
 12 CALL GWFN (J,I,P,IX) 
 13 CONTINUE 
    LAY=0 
    IF(NM<2)-KANf1)) 15,14,15 
 14 LFF=0 
    GO TO 17 
 15 IF(NM(2)-KAN(3)) 16,17,16 
 16 Nn0PS=0 
17 WRÎTEÎ3,100Î 
100 FORMAT('1',10X,'DATA')
    RETURN 
    END
```
Subroutine GWEN

This subroutine performs the transformations specified on the transformation cards. Special transformations can be performed using the variable subroutines JANET, LEANN, and SUZIE.

```
SUBROUTINE GWEN (J, I, P, IX)
     COMMON B(10,10), X(8,400), T(8,409), NM(80), JT(100),
    1 IT(100), PT(100), IXT(100), A(8), MN(75), NOOB S, NOVAR, LEE,
    2KFLAG, KAN(50)
     GO TO (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 19)120, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, IX
   1 DO 101 K=1, NOOBS
     IF(T(J,K)) 1000, 1000, 101
 101 T(J,X)=ALCG(T(J,K))RETURN
                                          2 00 102 K=1, NOOBS
     IF(T(J,K)) 1000, 1000, 102
 102 T(J,K)=ALCG10(T(J,K))
     RETURN
1000 WRITE(3,1001)
1001 FORMAT('0',10X,'****ARGUMENT NEGATIVE OR ZERO****')
     RETURN
   3 DO 103 K=1, NOCBS
 103 T(J,K)=SIM(T(J,K))RETURN
   4 DD 104 K=1, NOOBS
 104 T(J,K)=COS(T(J,K))
     RETURN
   5 DO 105 K=1, NOORS
105 T(J,K)= EXP(T(J,K))RETURN
   6 DO 106 K=1, NOOBS
106 T(J,K)=T(J,K)+T(I,K)
     RETURN
   7 DO 107 K=1, NOOBS
107 T(J,K)=T(J,K)+X(I,K)RETURN
   8 DO 108 K=1, NOOBS
108 T(J,K)=T(J,K)+P
     RETURN
  9 00 109 K=1, NOOBS
109 T(J,K)=T(J,K)-T(I,K)
     RETURN
```
10 DO 110 K=1, NOOBS $110 T(J,K)=T(J,K)-X(I,K)$ **RETURN 11 DO 111 K=1,N00BS 111 T(J,K)=T(J,K)-P RETURN 12 00 112 K=1,N00BS 112 T(J,K)=T(J,K)*T(I,K) RETURN 13 DC 113 K=1,N00BS** 113 T(J,K)=T(J,K) *X(I,K) **RETURN 14 DO 114 K=1,NQ0BS 114 T(J,K)=T(J,K)*P RETURN** 15 DO 115 K=1, NOOBS **115 T(J,K)=ABS(T(J,K)) RETURN 16 DO 116 K=1,N008S IF(T(I,K)) 116,1000,116 116 T(J,K)=T(J,K)/T(T,K) RETURN 17 DO 117 K=1,N00BS IF(X(I,KJ) 117,1000,117 117 T{J,K)=T(J,K)/X(I,K) RETURN 18 ÏF(P) 118,1000,118 118 DO 218 K=1,N00BS 218 T(J,K)=T(J,KÏ/P RETURN 19 DO 119 K=1,N00BS 119 T(J,K)=T(I,K) RETURN 20 DO 120 K=1,N00BS 120 T(J,K)=X(I,K) RETURN 21 DO 121 K=1,N00BS 121 T(J,K)=P RETURN 22 DO 122 K=1,N00BS IF(T(J,K)) 1000,122,122 122 T(J,K)=T(J,K)**T(I,K) RETURN 23 DO 123 K=1,N00BS IF(T(J,K)) 1000,123,123**

```
123 T(J,K)=T(J,K)**X(I,K) 
    RETURN 
 24 KP=P
```

```
TF(P-FL0AT(KP)) 324,124,324
```

```
124 DO 224 K=1,N00BS
```
 $\mathcal{L}_{\text{max}} = \mathcal{L}_{\text{max}}$

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```
224 T(J,K)=T(J,K)**KP 
    RETURN 
324 DO 424 K=1,N00BS 
     IF(T(JtK)) 1000,424,424 
424 T(J,K)=T(J,K)**P 
    RETURN 
 25 DO 125 K=2,N006S 
125 T(J,K)=T(J,K)+T(J,K-1) 
    RETURN 
 26 K=NOOBS 
126 T(J,K)=T(J,K)-T(J,K-1) 
    K=K-1 
    TF(K-l) 226,226,126 
226 T(J,1)=0.0RETURN 
 27 KP=N00BS-1 
    DO 127 K=1,KP 
    I=N008S-K-1 
127 T(J,I)={T(J,I)+T(J,I-l))/2.0 
    T(J,1)=0.0 
    RETURN 
 28 HUGE=10.0**48.0 
    KP=N00BS-1 
    DO 328 1=1,KP 
    P=HUGE 
    DO 228 L=I,NOOBS 
    IF(P-T(J,L)) 228,228,128 
128 P=T(J,L) 
    K=L 
228 CONTINUE 
    DO 328 L=l,8 
    P=X(L,I) 
    X(L,n = X( L,K) 
    X(L,K)=P 
    P=T(L,I) 
    T(L,I)=T(L,K) 
328 T(L,K)=P 
    RETURN 
 29 CALL JANET (J) 
    RETURN 
 30 CALL LEANN (J) 
    RETURN 
 31 CALL SUZIE (J) 
    RETURN 
    END
```
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Subroutines JANET, LEANN, and SUZIE

 \mathcal{A} and \mathcal{A} are the set of the set of the set of the \mathcal{A}

Variable subroutines JANET, LEANN, and SUZIE are included here as dummy subroutines (see page 325 of this Appendix).

```
SUBROUTINE LEANN (J) 
 COMMON B(10,10),X(8,400),T(8,409),NM(80),JT(100), 
lITdOOÏ ,PT(100) tlXTdOO) t A(8) ,MN(75) ,NOOBS,NOVAR,LEE, 
2KFLAG,KAN(50! 
 DUMMY=1 
 RETURN 
 END
```
Carl Committee

```
SUBROUTINE JANET (J) 
CCMHON B(10,10),X(8,400),T(8,409),NM(80),JT( 100), 
IIT(IOO),PT(100),IXT(100),A(8),MN(75),NOOBS,NOVAR,LEE, 
2KFLAG»KAN(50} 
DUMMY=1 
RETURN 
END
```

```
SUBROUTINE SUZIE (J) 
COMMON 8(10,10,X(8,400),T{8,409),NM(80),JT{ 100) , 
1TT(100),PT(100),IXT(100),A(8),MN(75),NOOBS,NOVAR,LEE, 
2KFLAG,KAN(50) 
nUMMY=l 
RETURN 
END
```
Subroutine KATHY

Subroutine KATHY prints out the current values of the transformed variables and the constants in the A array. This subroutine is used whenever a PRINT card is executed, and by subroutine NORMA when PACOCO or REGRESSION cards are executed.

```
SUBROUTINE KATHY 
    COMMON P(10,10),X(8,400),T(8,409),NM(80),JT(100), 
   IIT(IOO),PT(IOO),IXT(IOO),A(8),MN(75),NOOBS,NOVAR,LEE, 
   2KFLAC,KAN(50) 
    WRITE(3,100) (I, I=1, NOVAR)
100 FORMATAI»,• TRANSFORMED VARIABLES',//,'0 OBS», 8(13X, "T • , II ) ) 
    on 1 I=1,N00BS 
  1 WRITE(3,200) I,(TÎJ,I),J=1,N0VAR) 
200 F0RMAT(1X,I5,8E15.6) 
    WRITE(3,300) (T,A{I),I=1,8) 
300 FORMAT('0',/,10X,'A VALUES',8(/,20X,II,')',E15.6)) 
    RETURN 
    FNO
```
Subroutine NORMA

Using the previously defined notation where T1 is the dependent variable and T2, T3, ..., Tn are independent variables, $(n = number of variables, two to eight)$, subroutine NORMA determines the rectangular systems matrix, A/C , for subsequent use in the regression analysis. This matrix contains the normal equations used in the least squares method of estimating regression coefficients (71). The

first n columns contain the matrix A and the last column contains the matrix or vector C.

ΣTn ΣT1
ΣT2Tn ΣT2T1 NOOBS $\frac{\Sigma T2}{\Sigma (T2)}$ $\frac{\Sigma T3}{\Sigma T2}$ $\frac{\Sigma T}{\Sigma T}$
 $\frac{\Sigma T}{\Sigma (T3)}$ Σ T₂ ΣT2Tn ΣT2Tl
ΣT3Tn ΣT3Tl ZT3 Σ T2T3 Σ T3Tn \bullet $\ddot{}$ Σ (Tn)² Σ TnTl ZTn ET2Tn ET3Tn

NOOBS is the number of observations of the variables,

```
SUBP NUTTINE NORMA
   CHMMON R(10,10)tX(RtAOO)»T(8,409),NM(P0),JT(100Î, 
  11T(100), PT(100), IXT(100), A(8), MN(75), NOOBS, NOVAR, LEE,
  2KFLAG,KAN(50) 
   rn 1 1=1,10 
   no 1 j=i,io 
 1 P(I,J)=0.0N=N0VAP-1 
   no 5 M=l,NOOBS 
   DO 5 1=1,NOVAR 
   FACTP=T(I,M) 
   IF(I-l) 3,2,3 
 2 FACTP=1.0 
 3 J=I 
31 J=J+1 
   IF(J-NOVAR) 4,4,41 
 4 R(I,J)=B(I,J)+FACTR*T(J,MJ 
   CO TO 31 
41 R(I,NnVAP+l)=FACTR*T(1,M)+B(I,N0VAR+1) 
 B( I , I ) = T( T,M)#T( I ,M) + B(I ,T) 
   R{1,1 )=NOnRS 
   DO 6 1=1,N 
   K = I + 1no 6 J=K,NnVAR 
 6 p(j,n=B(i,j) 
   CALL KATHY 
   CALL CORA (NOVAR) 
   RETURN 
   FNO
```
Subroutine CORA

Subroutine CORA is called by subroutine NORMA to calculate the partial correlation coefficients between all possible pairs of variables. These are calculated using the equation (51)

$$
r_{ij} = \frac{N\Sigma T i Tj - \Sigma T i \Sigma Tj}{\sqrt{N\Sigma T i^2 - (\Sigma T i)^2] [N\Sigma T j^2 - (\Sigma T j)^2]}}
$$

where,

$$
r_{ij}
$$
 = partial correlation coefficient between variables
\nTi and Tj

 $N =$ number of observations of variables or NOOBS

$$
i,j =
$$
integer between 1 and the number of variables
(maximum of 8).

More specifically, these are zero order partial correlation coefficients; the correlation between any two variables is determined neglecting all other variables. This is opposed to higher order partial correlations which determine the correlation between variables while holding all other variables constant.

SUBROUTINE CORA (N) COMMON B(100), X(8,400), T(3272) WRITE(3,100) **100 FORMAT;;I«,22X,«PARTIAL CORRELATION COEFFICIENTS'?** $M = B(1)*B.0$ SYY=0.0 $DQ 1 I = 1, M, 8$

```
1 SYY=SYY+T(I) *T(I)
    NN = N * 10 + 1SYY=B(1)*SYY-B(NN)*B(NN)DD 4 I = 2, NK = I - 1M=K*10+II = NN + I - IB(91) = (B11)*B(II)-B(NN)*B(II)/SQRT(SYY*(B(1)*B(M)-B(I))1* B(1))
    IF(I-2) 4,4,2
  2 DO 3 J=2, K
    KK = (J-1)*10+JL = M + J - I3 B(J+90) = (B(1)*B(L)-B(J)*B(I))/SQRT((B(1)*B(M)-B(I)*D(J)))1B(I))^*(B(1)*B(KK)-B(J)**2))4 WRITE(3,200) I, (8(J+90), J=1, K)
200 FORMAT ('0', 1X, 18, F10.3, 6F8.3)
    WRTTE(3,300) (I, I=1, K)300 FORMAT ('0', 1X, 117, 618)
    RETURN
    END
```
Subroutine RENE

This subroutine is called whenever a REGRESSION card is executed. Subroutine RENE first calls subroutine NORMA to determine the rectangular systems matrix and partial correlation coefficients. Subroutine DONNA is then called to calculate the coefficients (B1, B2, ..., Bn) of the regression equation,

 $T1 = B1 + B2*T2 + B3*T3 + ... + Bn*Tn.$

The resulting regression equation is then used for each observation to calculate the estimated value, T1, of the

observed dependent variable, Tl, and the difference, Tl - $\hat{\tau}$ l, between observed and calculated values. This difference is also expressed as a percentage of the average of the observed and calculated values and as a percentage of the mean of all observed values of the dependent variable. The following quantities are also determined (53).

Root mean square of
$$
\sqrt{\frac{\sum (\text{8 difference})^2}{N - n}}
$$

Second difference
$$
\sum \frac{\sum (\text{71 - T1})^2}{N - n}
$$

estimate

and,

$$
100R = \frac{N\Sigma T1 \cdot \hat{T1} - \Sigma T1 \Sigma \hat{T1}}{\sqrt{\left[N\Sigma T1^2 - (\Sigma T1)^2\right] \left[N\Sigma \hat{T1}^2 - (\Sigma \hat{T1})^2\right]}}
$$
(100)

where,

 $N =$ number of observations, NOOBS n = number of variables, NOVAR lOOR = multiple correlation coefficient expressed in percent.

Finally, subroutine TERI is called to print out the matrices used in calculating the regression coefficients.

```
SUBROUTINE RENE
     CCMMON B(10,10), X(8,400), T(8,409), NM(80), JT(100),
    1IT(100), PT(100), IXT(100), A(8), MN(75), NOOBS, NOVAR, LEE,
    2KFLAG, KAN (50)CALL NORMA
     N = NQVARK = NQV AR + 1DC 1 J=1,KJJ=J+400DO 1 I = 1, N1 T(1, JJ) = B(I, J)WRITF(3,100)
100 FORMAT(/////////,1X, REGRESSION MODEL ----- T1 = 81 +',
   1' R2*T2 + R3*T3 + . . . . . + BN*TN',//)
     CALL DONNA (N,1)
     WRITE(3,200) (I, B(I,K), I=1,N)200 FORMAT('0',17X,'8',I1,'=',F20.8)
    WRITF(3,300)
300 FORMAT('1',2X,'OBS',3X,'CBSERVED T1',3X,'COMPUTED T1',
   15X, ID I F F E R E N C E', /,40X, 'ACTUAL', 9X,
   2'0/0', 8X, 12')
    Z = C_0 0
    DC 2 K=1, NOOBS
  2 Z = Z + T (1, K)YBAR=Z/(FLOAT(NOOBS))
    Z=0.0SY=0.0SYH=0.0SYY=0.
    SYHYH=0.0SYYH = C \cdot 0SCIFF=0.0DO 4 K=1, NO08S
    Y=T(1,K)YHAT=B(1,N+1)DQ 3 I=2, NOVAR
  3 YHAT=YHAT+B(I,N+1)*T(I,K)
    T(8,K)=YHATDIFF = Y - YHATSDIFF=SDIFF+DIFF*DIFF
    E=100.0*DIFF/YBAR
    P = 200.0*0IFF/(Y+YHAT)Z = 7 + P * PSY = SY + YSYH=SYH+YHAT
    SYY = SYY + Y * YSYHYH = SYHYH + YHA T * YHATSYYH = SYYH+Y*YHAT4 WRITE(3,400) K, T(1, K), YHAT, DIFF, P, E400 FORMAT(1X, I5, 2514.5, E14.4, F11.3, F10.3)
```
```
P = NQQRS - NSEE = (SDIFF/P)**0.5Z = (7/P) * *0.5P = NOOBSR=(P*SYYH-SY*SYH)/SQRT((P*SYY-SY*SY)*(P*SYHYH-SYH*SYH)
   1*0.00011WRITE(3,500) Z, SEE, R
500 FORMAT(*0*,///,1X,*ROOT MEAN SQUARE OF PERCENT DIFFER*,
   I'ENCES', FI8.3,///, IX, 'STANDARD ERROR OF ESTIMATE', 13X,
   2F18.3, 7777, 18.7 R = MULTIPLE CORRELATION COEFFICIENT
                                                            \mathbf{R}3' 100 (R) = , 58.3, 77, 68, 12 = 100(DIFF)/(MEAN OBSERV^2)4!FD T1)')
    CALL TERI (N, 1)
    RETURN
    END
```
Subroutine DONNA

Subroutine DONNA determines the regression coefficients (Bl, B2, ..., Bn) along with the inverted matrix, A^{-1} , of the matrix A found in subroutine NORMA. The invert of matrix A. A^{-1} . is found by using the abbreviated Doolittle method (53, 71). The solution vector, B, containing the regression coefficients, is then equal to the product of matrix A^{-1} and vector C, (from subroutine NORMA).

```
SUBROUTINE DONNA (N,M)
  CEMMON R(10,10)
  N M = N + MNMI=NM+1NI=N+1N2 = N + 2DO 1 K=1,NB(N2, K) = K1.8(K, NM1)=K00101 R=1, NK = N + 1 - I RBIO=0.000 4 I=1,KDD 4 J=1,K
```

```
IF(ABS(B(I, J)) - BIG) 4,4,3
3 RIG=ABS(B(I,J))
   L = IM = J4 CONTINUE
  DD 5 I=1, N2TFMP=B(I,1)B(I, 1) = B(I, MM)5 B(I,MM)=TEMPDO 6 J=1, NM1
   TEMP=R(1,J)B(1, J) = B(L, J)6 B(L, J)=TEMP
   B(N1, NM)=1.0/B(1,1)D0 7 K=2, NM
7 B(N1, K-1)=B(N1, NM) * B(1, K)
   TFMP=B(1, NM1)BIG=B(N2,1)DQ 9 I = 2, NIM1=I-1DO 8 J=2, NM
 8 B(IM1, J-1)=B(I, J)-B(I, 1) *B(N1, J-1)
   B(IM1, NM) = -B(1, 1) * B(N1, NM)B(IM1, NM1)=B(1, NM1)9 R(N2, IM1)=B(N2, I)R(N, NM1) = TEMPB(N2, N) = BIGDO 10 K=1, NM
10 B(N,K)=R(N1,K)nn 11 K=1 NTEMP = B(K, NM!)B(K, NM1) = B(N2, K)11 B(N2,K)=TEMPDO 13 K=1, M
   DC 13 I=1,N
   TEMP = R(I, 1)DC 12 J=2, NM
12 B(I,J-1)=R(I,J)13 B(I, NM)=TFMPM N = N - 1DO 16 I = 1, MMX I = IDO 14 J=1 NIF(R(N2, J)-XI) 14,15,14
14 CONTINUE
15 DO 16 K=1,N2TEMP=B(K, I)B(K, I) = R(K, J)
```

```
16 B(K, J) = TEMP
```

```
DO 19 1=1,MM 
   XI = Ino 17 J=1,N 
   IF(B(J, NM1)-XI) 17,18,17
17 CONTINUE 
18 DO 19 K=1,NM1 
   TEMP=R(I,K) 
   P(I,K)=P(J,K)19 B(J,K)=TEMP
   RETURN
   F NO
```
Subroutine TERI

This subroutine is sued to print out the matrices used in the regression analysis. First, the rectangular system's matrix, A/C, as calculated by subroutine NORMA, is printed. The identity or unit matrix, I, is then calculated as the matrix product, $A^{-1}A$. All of the elements of the identity matrix should be zero except for the elements in the principal diagonal which should be one. The identity matrix and solution vector, B, are printed out as matrix I/B. Finally, matrix A^{-1} is printed out with the solution vector.

```
SUBROUTINE TERI (N,M)
    COMMON R(10,10),X(8,6001,T(8,409),NM(80),JT(IOO), 
   1TT(10G),PT(100),TXT(100),A(8),MM(75),NOOBS,NOVAR,LEE, 
   2KFLAG,KAN(50) 
    WPTTF(3,100) 
100 C0RMAT(«1',30X,'MATRIX ECUATION•,5X,•A(B)=C',///,LOX, 
   1'RECTANGULAR SYSTEMS MATRIX»,5X,•(A/C)•) 
    K = N + ML = 400 + Kno 1 I=1,N1 WPITE(3,200) (T(I,J),J=401,L)
```

```
200 FORMAT ('0', 9E13.5)
     WRITE(3,300)
300 FORMAT('0',///,' IDENTITY AUGMENTED WITH SOLUTION ',
    1'VECTOR(S) (1/B)<sup>*</sup>)
     L = N + 1\sim 10^{-11}DG 4 I=1, N\mathcal{L}^{\text{max}}_{\text{max}} , where \mathcal{L}^{\text{max}}_{\text{max}}D0 3 J=1,KKK = 400 + JC = 0 = 0
     DO 2 LL=1,N2 C=C+R(I,LL)*T(LL,KK)
  3 R(L, J) = C\sim 10^{11} km s ^{-1}4 WRITE(3,400) (B(L, J), J=1, K)400 FORMAT ('0', 9F13.6)
     WPITE(3,500)
500 FORMAT('0',///,1X,'INVERTED RSM MATRIX (.A./B)')
     DD 5 I = 1, N\sim 10^75 WRITE(3,200) (B(I,J),J=1,K)
     WRITE(3,600)
500 FORMAT('1')
     RETURN
     FND
```
 \sim ϵ \sim

 $\sim 10^{-1}$

 $\sim 10^{11}$

Example input and output

The major use of MAIDS at Iowa State University has been for the determination of prediction equations for filter cake resistance indices or β indices. The general form of the β prediction equation is:

$$
\beta = 10^{b_1} \, c_S^{b_2} \, c_F^{b_3} \tag{33}
$$

which can be made linear by a logarithmic transformation; i.e., log $\beta = b^1 + b^0$ log $C_g + b^1$ log C_g . MAIDS can therefore be used to determine the coefficients b_1 , b_2 , and b_3 .

If data are collected with C_S constant and the assumption is made that β varies only with the ratio C_S/C_F (i.e., $b_2 = -b_3$ in Equation 33), then the following prediction equation may be used:

$$
\beta = 10^{\text{b}} \text{1} \left(\frac{c}{s} / c_{\text{F}} \right)^{\text{b}} \tag{15}
$$

MAIDS can again be used to determine the coefficients b_1 and $b_2 \cdot$

The following examples give the input and output for three MAIDS jobs. The first two examples involve the determination of **B** prediction equations. The **B** prediction equations are also calculated manually for comparison with the computer results. The third example shows the use of MAIDS for problems other than regression analysis.

BO COLUMN DATA SHEET

 $\mathcal{L}(\mathcal{A})$ and $\mathcal{L}(\mathcal{A})$. The set of $\mathcal{L}(\mathcal{A})$

 $\mathcal{L}^{\text{max}}_{\text{max}}$, $\mathcal{L}^{\text{max}}_{\text{max}}$

 \mathcal{L}

 ~ 100 km s $^{-1}$

 $\langle \cdot \rangle$

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2.$

80 COLUMN DATA SHEET

 357

 $\mathcal{A}^{\mathcal{A}}$

 $\ddot{}$

 \sim \sim

 $\mathcal{L}^{\text{max}}_{\text{max}}$

80 COLUMN DATA SHEET

MAIDS DILLINGHAM 1964--BESETT 1966--BRIDGES 1967

 \mathbf{C} EXAMPLE NUMBER 1

DATA FOLLOW

 C

 $\mathbf C$

 \mathbf{C}

 \mathbf{C}

 \mathbf{C}

 \mathbf{c}

 \mathbf{C}

L RUN NO.

 $21.$

 $22 -$

 $23.$

 $24.$

 $25.$

 $26.$

 $27.$

 $34.$

 $25.$

 $36.$

 $37.$

 $38.$

 $29.$

WATER PLUS FERRIC CHLOPIDE

BETA/F04 = BETA DIVIDED BY 10000

CELITE 535 FILTER AID

 CS^-

7.85

 8.16

 8.27

 $8 - 26$

 8.26

 8.26

 9.36

 7.57

7.73

 3.54

 3.36

 2.59

 3.56

360

DETERMINATION OF SETA INDEX PREDICTION EQUATION

FOR THE FILTRATION OF UNIVERSITY TAP

BRIDGES AND ARORA FILTER RUNS 21-27 AND 34-51

 CF

 $400 -$

 $160.$

 $400.$

 $320.$

 $266.$

 $230.$

 $400 -$

 $266.$

400.

200.

100.

 $133.$

 $200.$

BETA/E04

8200.

710.

 $990.$

1970.

 $2560.$

895.

1690.

 $650.$

775.

 $4650.$

 $3000 -$

765.

543.

 $\Delta \sim 10^4$

 \sim .

 $\hat{\mathcal{L}}$

 $\mathcal{L}(\mathcal{M})$.

TRANSFORMED VARIABLES

A VALUES

PARTIAL CORRELATION COEFFICIENTS

- $2 -0.245$
- $3 -0.697$ 0.856 (i.e. $r_{12} = -0.245$) $1 \t 2$

363

 $B2 =$ 1.71041584

 $B3=$

 -2.12593842

R = MULTIPLE CORRELATION COEFFICIENT $100(R) = 97.325$

 $Z = 100(DIFF)/(MEM \tBSERVED \tTI)$

MATRIX EQUATION A(B)=C

 $\sim 10^7$

المتسجد ويستنبذ والمتحدث

IDENTITY AUGMENTED WITH SOLUTION VECTOR(S) (I/B)

INVERTED RSM MATRIX (.A./B)

 $\mathcal{A}(\mathcal{A})$ and $\mathcal{A}(\mathcal{A})$.

DATA

END

 $\sim 10^7$

 ~ 100 km s $^{-1}$

MAIDS DILLINGHAM 1964—BESETT 1966—BRIDGES 1967

- C EXAMPLE NUMBER 2
- C DETERMINATION OF BETA PREDICTION EQUATION
- C FOP THE FILTRATION OF UNIVERSITY TAP
- C WATER PLUS FERRIC CHLORIDE
- C CS HELD CONSTANT
- C SIL-FLO 443 (BAG 2) FILTER AID
- C ARORA FILTER RUNS 116-121

T *10000.

T LOG LOG LOG

 $T \rightarrow T3$

V* 2

REGRESSION

TRANSFORMED VARIABLES

A VALUES

PARTIAL CORRELATION COEFFICIENTS

REGRESSION MODEL ----- T1 = 81 + B2*T2 + B3*T3 + + BN*TN

 $\mathbf T$

 \mathcal{L}^{\pm} and \mathcal{L}^{\pm}

 $R = MULTIPLE CORRELATION COEFFICIENT 100(R) = 99.167$

 $Z = 100$ (DIFF)/(MEAN OBSERVED T1)

MATRIX EQUATION A(B)=C

RFCTANRULAR SYSTEMS MATRIX (A/C) 0.60000F 01 -0.82965E 01 0.45?94E 02 $-$ C.82965E 01 0.11761E 02 $-$ 0.62198E 02

IDENTITY AUGMENTED WITH SOLUTION VECTOR(S) (I/B)

INVERTED RSM MATRIX (.A./B)

0.67751F 01 0.477925 01 0.1C296E 02

0.47792E 01 0.34563E 01 0.19749E 01

 $\mathcal{A}_{\mathbf{q}}$, and $\mathcal{A}_{\mathbf{q}}$, and $\mathcal{A}_{\mathbf{q}}$

DATA

END

MAIDS

368

 \sim

 \mathcal{A}

 369

TRANSFORMED VARIABLES

 $\mathcal{L}^{\text{max}}_{\text{max}}$ and $\mathcal{L}^{\text{max}}_{\text{max}}$

A VALUES

DATA

 $A = 5.0$ 7.0 -3.0

 $\ddot{\bullet}$

 $\mathcal{L}^{\text{max}}_{\text{max}}$, $\mathcal{L}^{\text{max}}_{\text{max}}$

370

 $\sqrt{2} \, \frac{1}{2} \, \frac{1}{2$

TRANSFORMED VARIABLES

A VALUES

DATA

STOP

Manual calculations

Example number 1

Bridges and Arora Filter Runs 21-27 and 34-51 Filtration of University tap water plus ferric chloride

Celite 535 filter aid

The data from these filter runs are printed out as part of the computer output (pp. 360-361). Since filter runs were made with both C_S and C_F varied, a prediction equation of the form of Equation 33 can be determined.

Calculations Log β versus log C_F is plotted for each value of C_S as shown in Figure 52. The slope of these plots is equal to b_2 . From Figure 52:

 $b_3 = -2.14$

Values of β at each value of C_S can be determined from Figure 52 at a selected value of C_p . In this case the following values were determined for $C_p = 300$ mg/l:

Figure 52. Log log plot of β versus C_p for the data in Example 1

 \bar{z}

 \sim

 \mathcal{L}^{\pm}

 $\mathcal{A}^{\mathcal{A}}$

 \mathcal{A}^{c} and

 \sim

 \sim ω

A plot of log β versus log C_S at C_F = 300 mg/l is shown in Figure 53. The slope of this plot is equal to b_2 and from Figure 53:

$$
b_2 = 1.75
$$

Thus :

$$
\beta = 10^{b_1} \text{ C}_S^{1.75} \text{ C}_F^{-2.14}
$$

and from Figure 53, $\beta = 21.2 \times 10^6$ ft⁻² when C_S = 10 mg/l and $C_F = 300$ mg/1. Therefore:

$$
21.2 \times 10^6 = 10^{b}1 \cdot 10^{1.75} \cdot 300^{-2.14}
$$

and taking the logarithm of both sides of the equation:

$$
7.32634 = b_1 + 1.75(1) - 2.14(2.47712)
$$

and

$$
b_1 = 10.87738
$$

Therefore, the final result is:

$$
\beta = 10^{10.88} \text{ c}_S^{1.75} \text{ c}_F^{-2.14}
$$

The prediction equation determined by using the computer is :

$$
\beta = 10^{10.88005} \text{ C}_S^{1.701042} \text{ C}_F^{-2.12594}
$$

Figure 53. Log log plot of β versus C_S for the data Example 1

 $\sim 10^{-10}$

Example number 2

Arora Filter Runs 116-121 Filtration of University tap water plus ferric chloride Sil-Flo 443 (Bag 2) filter aid

The data from these filter runs are printed as part of the computer output (p. 365). During these filter runs the iron concentration was held constant. It will be assumed that there are no concentration effects so that a prediction equation of the form of Equation 15 can be used.

Calculations From the plot of log ß versus log C_S/C_F shown in Figure 54, the value of b₂ can be determined. In this case:

$$
b_2 = 1.97
$$

and from the figure, when $C_S/C_F = 0.04$, β is equal to 3.38 \times 10⁷ ft⁻². Therefore:

$$
3.38 \times 10^{7} = 10^{b_1} (0.04)^{1.97}
$$

and by taking the logarithm of both sides of the equation :

7.52892 =
$$
b_1
$$
 + 1.97(-1.39794)
Therefore:

 $b_1 = 10.28286$ and the final result is : 377

Figure 54. Log log plot of β versus C_S/C_F for the data from Example 2

 $\sim 10^{11}$ km $^{-1}$

 \mathcal{L}_{max} , \mathcal{L}_{max}

 $\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L$

$$
\beta = 10^{10.28} (c_S/c_F)^{1.97}
$$

 \mathcal{L}^{\pm}

 \bar{z}

The prediction equation determined by using the computer is:

$$
\beta = 10^{10.29649} (C_S/C_F)^{1.97489}
$$

APPENDIX F

POPO Program User Manual

Introduction

The Program for Optimization of Plant Operation or the POPO Program was designed to read in all of the necessary filtration data and cost information and to compute the total filtration cost for all desired combinations of filtration rate, terminal head loss, and body feed rate. The 10 combinations which result in the lowest total costs are printed out for 8 indices equal to 50, 75, 100, 125, 150, **and 175** percent of those predicted by the S prediction equation. The method of calculating total cost is identical to that outlined on pages 161-171.

POPO can be used to optimize the design of proposed filtration plants or to optimize the operation of existing plants. For an existing plant, the filter area and therefore the filtration rate are fixed. POPO can then be used to determine the combination of terminal head loss and body feed rate which can be used to give the lowest operating costs for the plant.

Any number of POPO jobs can be processed in one computer run and, in successive jobs, one or more items of the input data can be changed. Input data that are not changed remain the same as in the preceding job. This enables the comparison of different types and grades of filter aids, different

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types of filters, different influent water qualities, etc.

Input card formats

Data used by POPO is read into the computer on 80 column punched cards. The card formats used with POPO are shown in Table 36.

The first input card shown in Table 36 must always be the first card read by the computer. If this card is not present, the computer run is ended for the information on this card is used to determine the values of data on the other input cards. This card should not be repeated if more than one POPO job is processed in the same computer run.

All other input data cards are designated by an index number punched within columns 1 to 5. The index number determines what type of information is contained on the card. Values of data on these cards must be punched anywhere within columns 26 to 50. Columns 7 to 25 and 51 to 80 are reserved for labeling the data and are ignored by the computer. The individual data cards are explained below by index number.

0. Comment card. Any desired comments can be made within columns 7 to 80. Additional comment cards may be placed anywhere within the deck of input data cards. All comment cards must have G punched within columns 1 to 5.

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Table 36. POPO input card forms

 Δ

 $\label{eq:2} \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^{2} \left(\frac{1}{\sqrt{2}}\right)^{2} \left(\frac{1}{\sqrt{2}}\right)^{2}$

 \mathcal{A}

383

 $\ddot{}$

 \bar{a}

Table 36 (Continued)

 $\mathcal{L}^{\text{max}}_{\text{max}}$

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2.$

1. The design flow in MGD.

2. The salvage value of the plant at the end of its design life in percent of the first cost.

3. The overall pumping efficiency in percent.

4. The annual interest rate in percent.

5. The expected life of the plant in years.

6. The suspended solids concentration in mg/l or ppm for use in predicting the β index.

7. The ξ index of the filter aid in ft/lb. The ξ index may be written in exponential form by placing the letter E before the exponent of 10 (i.e. 1.95E9 = 1.95 x 10^9).

8. The temperature of the filter influent in degrees Fahrenheit.

9. The weight of the precoat, w, in Ib/sq ft.

10. The in-place bulk density of the clean filter aid, γ_n , in lb/cu ft.

11. The outer diameter of the filter septa in inches. Use 0 if flat septa are used.

12. The exponents of the β prediction equation. The general form of 6 prediction equation used in POPO is:

$$
\beta = 10^{b} \, \text{C}_{\text{S}}^{b} \, \text{C}_{\text{F}}^{b} \, \text{S} \tag{33}
$$

An additional exponent, $b_{\hat{A}}$, is included on the POPO input card in case it is desired to add an additional variable, such as ξ , to the prediction equation. When Equation 33 is

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used, b_A equals 0. The values of the exponents are separated by slashes (/).

13. The values of the initial, increment, and final filtration rates in gpm/sq ft. Adjacent values are separated by a slash $($).

14. The values of the initial, increment, and final body feed rates in mg/1 or ppm. Adjacent values are separated by a slash $($).

15. The values of the initial, increment, and final terminal head losses in ft of water. Adjacent values are separated by a slash (/).

16. The cost of the filter aid in \$/ton.

17. The card with index number 17 is followed by cards defining the first cost versus area curve for the plant. Each of these cards contains a value of filter area in sq ft and the corresponding first cost in \$/sq ft. These cards must be arranged in ascending order according to the filter area. Up to 50 first cost versus area cards may be read in and the last card must have an asterisk (*) punched in Column 6.

18. The cost of electrical power in $\frac{c}{kwh}$.

19. The card with index number 19 is followed by cards defining the labor and maintenance cost versus area curve for the plant. Each of these cards contains a value of filter area in sq ft and the corresponding labor plus

386
maintenance cost in \$/sq ft per month. These cards must be arranged in ascending order according to the filter area. Up to 50 labor plus maintenance cost versus area cards may be read in and the last card must have an asterisk (*) punched in column 6.

20. The amount of water required to backwash the filter in gal/sq ft and the length of time required to backwash and precoat the filter in minutes.

21. Card index number 21 is reserved for inputing maintenance cost data. In the present form of POPO, labor and maintenance costs are combined so that cards with index number 21 are not used.

The cards containing all of the necessary input data for each POPO job are followed by the BEGIN card. This card indicates to the computer that all of the input data have been read in and the optimization calculations can now be made. The B of the BEGIN card must be punched in column 6.

After the optimization calculations for a POPO job are completed and the results output, the computer will begin to read in the data for another POPO job. To end a computer run, a STOP card should be placed after the BEGIN card for the last POPO job to be processed. The S of the STOP card must be punched in column 6.

FORTRAN listing

POPO was originally written by Dillingham (27) in FORTRAN II computer language (40) for use with the IBM 7074 computer system. The program has been converted to FORTRAN IV computer language (31, 41) for use with the IBM 360/65 computer system at Iowa State University and the FORTRAN listing for the program is given in this section. The major changes in POPO necessary to convert from FORTRAN II to FORTRAN IV are as follows:

1) FUNCTION subprograms

In FORTRAN IV a FUNCTION subprogram cannot contain a SUBROUTINE statement or another FUNCTION statement. Therefore, it was necessary to convert functions VALUE, VALU, and PRED to subroutines.

2) Alphameric code

When a character is input under an A format, it is stored as an integer value. For example, the letter B was stored in FORTRAN II as 6200000000. This value was used in subroutine READR to determine if column 6 of a data card contained the letter B. Such information was also used in function VALU to read numeric data.

Different FORTRAN systems have different alphameric codes. POPO has been changed to account for these changes by inputting the following array:

Characters read under an A format are then determined by comparison with elements of the above array. This array must be read in on the first data card. If it is not, the program is stopped.

3) Library functions

FORTRAN II library functions (i.e. SQRTF (X)) have been changed to FORTRAN IV (i.e. SQRT (X)).

The only other major change from Dillingham's version of POPO is the method used to account for the costs ascribed to backwashing. Dillingham (27) assumed that the increase in monthly costs resulting from providing filtered backwash water was equal to the total monthly operating cost times the ratio of the quantity of backwash water needed per month divided by the quantity of finished water produced per month. He also assumed that the increase in monthly costs due to down time for backwashing and precoating was equal to the monthly operating cost (excluding power costs) times the ratio of the down time per filter run to the length of the filtering cycle. Apparently, these assumptions were made to avoid the iterative procedure for calculating filter area. However, since the total operating cost includes the increase in cost due to backwashing, it was still necessary for Dillingham to use an iterative procedure to calculate backwashing costs.

Dillingham's method of calculating filtration costs may be valid when the filter run length is relatively long. However, the filter run length calculated in Dillingham's

version of POPO did not include the time required to backwash and precoat the filter. This error increased the calculated number of filter runs per month and therefore caused the monthly cost of precoat filter aid to be high. This in turn caused the monthly costs due to backwashing to be too high. Considerable error could result if the filter run length was short.

A complete explanation of the FORTRAN listing for POPO would be very lengthy. Therefore, only a brief explanation of the basic purposes of the main program and each subroutine is given. Some of the symbols used in FORMAT statements for the Iowa State University IBM 360/65 computer system have been explained in Appendix D. Figure 55 is a schematic diagram showing the relationships between the various subroutines of POPO. The arrows in Figure 55 point to the subroutine which is called. Input subroutines are herein defined as subroutines which are used for reading and interpreting the input cards. Operation subroutines are defined as subroutines used to perform the specified operations determined by the input subroutines. The computation of each cost factor is made in a separate subroutine so that any changes in the method of computing a particular cost factor can be easily made.

A summary of the notation used in POPO is listed in Table 37.

 $\frac{1}{4}$

Figure 55. Schematic diagram of POPO subroutines

 $\mathcal{L}^{\text{max}}_{\text{max}}$ and $\mathcal{L}^{\text{max}}_{\text{max}}$. The set of $\mathcal{L}^{\text{max}}_{\text{max}}$

 \sim

 $\mathcal{L}^{\text{max}}_{\text{max}}$

 $\sim 10^{11}$

 $\frac{1}{2}$

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2\pi i}}\sum_{j=1}^n\frac{1}{\sqrt{2\pi i}}\sum_{j=1}^n\frac{1}{\sqrt{2\pi i}}\sum_{j=1}^n\frac{1}{\sqrt{2\pi i}}\sum_{j=1}^n\frac{1}{\sqrt{2\pi i}}\sum_{j=1}^n\frac{1}{\sqrt{2\pi i}}\sum_{j=1}^n\frac{1}{\sqrt{2\pi i}}\sum_{j=1}^n\frac{1}{\sqrt{2\pi i}}\sum_{j=1}^n\frac{1}{\sqrt{2\pi i}}\sum_{$

 ~ 10

Table 37 (Continued)

 $\langle \cdot, \cdot \rangle$

 \mathcal{A}_{eff} .

 \sim \sim

Table 37 (Continued)

Name	Meaning
VIS	Kinematic viscosity, ft^2/hr
w	Weight of precoat, w, $1b/sq$ ft
ΧI	ξ index, ft/lb
XLP	Thickness of precoat layer, ft

MAIN program

The array contained on the first data card is read in. If this array is missing, the program is stopped. Subroutine READR is then called to read all of the data cards for the first job. Costs are then computed for all combinations of filtration rate, body feed rate, and terminal head loss for values of β equal to 50, 75, 100, 125, 150, and 175 percent of the predicted value. After the 10 least cost combinations are printed out for each value of β , control is transferred to statement 1 and subroutine READR is called to read the data cards for the next job.

COMMON IN(153 J,ANS(13,101,6(4),AMORT,AREA,BETA,BWT,CD,

```
ICDI,CDS,CDF,CDE,CPD,CF,CL,:M,CB,COPER,CTOTL,OS,EFF, 
     2FACTR,G,GP,GW,HP,HC,KPIT,KP0T,PHI,3I,QS,QF,QGPM,QMGD, 
     3QMGDP,RF,RO,RS,SIGMA,TH,THI,THS,THF,TR,THICK,UQ,VIS, 
     4W,XI,XLP 
      KPIT=1 
      KP0T=3 
      READ(<PIT,100)(IN(I),I=81,97),K 
  100 FDRMAT(10X,17A1,I5) 
C 
C THE FOLLOWING MUST BE THE FIRST DATA CARD READ 
           C 1234567890+-.E BS12345 
C 
      IF(K-12345)1000,1,1000 
    1 CALL READR 
      DO 9 MM=50,175,25 
      FACTR = FLOAT(MM)/100.0 
      UQ=QI-QS 
    2 UQ=UQ+QS 
      IF(UQ-QF)3,3,8 
    3 CD=CDI-CDS 
    4 CD=CD+CDS 
      IF(CD-CDF)5,5,2 
    5 TH=THI-THS 
    6 TH=TH+THS 
      IF(TH-THF)7,7,4 
    7 CALL DIEQS 
      CALL COSTS 
      CALL STRES 
      GO TO 6 
    8 CALL OUTPT 
    9 CONTINUE 
      GO TO 1 
 1000 STOP
```
C MAIN PROGRAM — POPO

END

 $\bar{\epsilon}$

Subroutine READR

After the ANS array is initialized with large numbers, a data card is read and the information on it is stored as elements 1 to 80 of the IN array and also printed out on the output sheet. If a BEGIN card is read (indicated by the letter B in column 6 so that $IN(6) = B$, the amortization factor is calculated, output sheet headings are written, and control is returned to the MAIN program. If a STOP card is read (indicated by the letter S in column 6), the program is stopped. If the card is not a BEGIN or STOP card, subroutine VALUE is called to determine the value of the index number in columns 1 to 5. Cards with index number 0 are ignored, otherwise control is transferred to the statement number corresponding to the index number of the card. Subroutine VALUE is called to interpret the data within columns 26 to 50 of the cards with index numbers of 1 to 15. For cards with index numbers of 16 to 21, subroutines CFUST, CPOWR, CLABR, CBAKW, and CMAIN are called to interpret first, power, labor, backwashing, and maintenance cost data, respectively.

After the data on the card has been interpreted, control is transferred to statement 51 and the next card is read.

```
SUBROUTINE READR 
     COMMON IN(150),ANS(13,10),B(4),AM0RT,AREA,BETA,BWT,CD, 
   1CDI,CDS,CDF,CDE,CP0,CF,CL,CM,CB,COPER,CTOTL,CS,EFF, 
   2FACTR,G,GP,3W,HP,HC,KPIT,KP0T,PHI,QI,QS,QF,QGPM,QMGO, 
   30MGDP,RF,RO,RS,SIGMA,TH,THI,THS,THF,TR,THICK,UQ,VIS, 
   4W,XI,XLP 
     WRITE{KPOT,IOO) 
100 FORMAT('1POPJ -- PROGRAM FOR OPTIMIZATION OF PLANT',
   1' OPERATION',///) 
    BIG=1000000.**4 
     DO 50 L=l,10 
 50 ANS(8,L)=BIG 
     6=32.2*3600.0*3600.0 
     GW=62.4 
 51 READ(KPIT,200)(IN(I ),I=1,80) 
200 F0RMAT(80A1) 
    WRITE(KPOT,300)(INC I ),1 = 1,80) 
300 FORMATC1X,80A1) 
    IF(IN(6)-IN(96))53,52,5352 Fl={l.+RATEI)**YRS 
    AMDRT=<RATEI/(F1-1.))*(Fl-PCT/100.)/12. 
    WRITECKPOT,400) 
400 FORMAT('1','FLOW TERM CF BETA TIME AREA ',<br>1'THICK # dee COSTS & REP MILLION CALLONS == # 1.
   1'THICK * --- COSTS, $ PER MILLION GALLONS -- *
   2'TOTAL',/,7X,'HEAD',9X,'4 -2',20X,'*',20X,'LAB+',13X,<br>3'*',4X,'COST',/,' GSFM FT PPM 10 FT HR ',
   3'*1,4X,*COST*,7,4S. GSFM FT PPM 10 FT HR *,<br>4'SO FT IN * TOTAL 1ST OPER MAIN POWR FAID',<br>5' * $/MO',//,1X,44('-'),'*',37('-'),'*',8('-'))
            5/M0',//,lX,44('-'),'*',37('-'),'*',8('-'))
    RETURN 
 53 IF( INC6)-IN(97))54,63,54 
 54 CALL VALUE (1,PP) 
    INDEX=PP 
    IF(INDEX)51,51,55 
 55 IF( INDEX-21) 56,56,51 
 56 30 TO (1,2,3,4,5,6,7,8,9,13,11,12,13,14,15,16,17,18, 
   119,23,21),INDEX 
  1 CALL VALUE (26,QMGD) 
    GO TO 51 
  2 CALL VALUE (?6,PCT) 
    50 TP 51
  3 CALL VALUE (26,EFF) 
    EFF=FFF/100. 
    GO TO 51 
  4 CALL VAL JE (?6,RATEI) 
    RATFT=9ATFI/100. 
    GO TO 51 
 5 CALL VALUE C26,YRS) 
    30 T1 51 
 6 CALL VALUE (26,CS)
```

```
30 TO 51
```
7 CALL VALUE (26,XI) SO TO 51 8 CALL VALUE (26,FTEMP) VTS=VISC0(FTEMP)*3600. 3D TO 51 9 CALL VALUE (26,W) GO TO 51 10 CALL VALUE (26,GP) 30 T3 51 11 CALL VALUE (26,8S) $RS = 2S/24.$ 30 TO 51 12 CALL VALUE (26,6(1)) CALL VALUE(0,3(2)) CALL VALUE(0,B<3)) CALL VALUE(0,B(4)) SO TH 51 13 CALL VALUE (26,31) 01=31*8.02 CALL VALUE(0,QS) $25=25*8.02$ IF(3S)58,58,57 57 CALL VALUE(0,QF) 0F=3F*8.02 **GO TH 51** 58 3S=1. $QF = QI$ SO TO 51 14 CALL VALUE (26,COT) CALL VALUE(0,CDS) IF(COS)53,63,50 59 CALL VALUE(0,CDF) SO TO 51 60 CDS=1. CDF=CDI GO T3 51 15 CALL VALUE (26,THI) CALL VALUE(3,T4S1 IF(THS)52,62,61 61 CALL VALUE(3,THF) 30 TO 51 62 THS=1. THF=T4I GO TO 51 16 CALL CDIAT(l) GO TO 51 17 CALL CFUST(L) 30 TO 51 18 CALL CPOWP(l) GO TO 51

```
19 CALL CLABR(l) 
   GO TO 51 
20 CALL CBAKW(l) 
   GO TO 51 
21 CALL CMATN(l) 
   GO TO 51 
63 STOP 
   END
```
Subroutine VALUE

This subroutine interprets the data on the input card which was read in as elements 1 to 80 of the IN array by subroutine READR. Subroutine VALU is called to convert data in alphameric form in the IN array to numeric form. The argument N is the element of the IN array, or column of the input card, at which the conversion process in subroutine VALU begins, and V is the value of the data in numeric form determined by subroutine VALU. N is set equal to 1 in subroutine READR when the index number in columns 1 to 5 is to be determined and to 26 when the first data value in columns 26 to 50 is to be determined. To determine additional data values in columns 25 to 50 (i.e. cards with index numbers 12, 13, 14, 15, 17, 19, and 20)» N is set equal to 0 and the conversion process in subroutine VALU then begins at column IN(100) which is the number of the column immediately following the previously determined value.

The letter E punched in the column immediately follow-

ing a number indicates that the number is written in exponential form. Subroutine VALU is called to determine the exponent of 10, EX, and the number is multiplied by 10^{EX} .

SUBROUTINE VALJE(N,V) COMMON IN(150) CALL VALU (N, V) **M=IN(100)** IF(IN(M)-IM{94))2,1,2 **1 CALL VALU (M,EX) V=V*10.**EX** 2 RETURN END

Subroutine VALU

This subroutine is used to convert data in alphameric form to numeric form. This is done by starting at the column indicated by N (see subroutine VALUE) and then checking each successive column until the number is reached. The number is then converted from alphameric to numeric form by comparing each digit of the number to the elements of the array read in on the first data card. If there is no number present, a value of 0 is returned.

SUBROUTINE VALU(N,VA) COMMON IN(150) LAST=50 IF(N)1,1,2 1 M=IN(100) GO TO 3 2 M=N 3 KVA=0 VA=0. KSN=I KD=C $IF(M-LAST)4,4,8$ 4 DO 7 KPOS=M,LAST IF(IN(KP0S)-IN(95))5,7,5 5 DO 6 K=81,93 $IF(IN(KPOS)-IN(K))6,13,5$ 6 CONTINUE 7 CONTINUE IN(100)=LAST 8 RETURN 9 KP0S=KP0S+1 IF(KPGS-LAST)11,11,10 10 KPOS=LAST GO TO 21 11 DO 12 K=81,93 IF(IN(KPOS)-IN(K))12,13,12 12 CONTINUE GO TO 21 13 J=K-8C GO TO (14,14,14,14,14,14,14,14,14,17,9,18,19),J 14 KVA=KVA*10+J 15 IF(<0)16,9,16 16 KD=10*KD GO TO 9 17 KVA=<VA*10 GO TO 15 18 KSN=-1 GO TO 9 19 KD=1 20 GO TO 9 21 VA=KVA*KSN IF(<D)23,23,22 22 VA=VA/FLOAT(KD) 23 IN(100)=KPOS RETURN FNP

 $\tilde{}$

Function VISCO

This subroutine calculates the value of the kinematic viscosity in ft^2 /sec from the temperature in $\textdegree F$. The argument C is the water temperature in °F. The kinematic viscosity is converted to ft^2/hr (VIS) in subroutine READR.

FUNCTION VISCO(C) VISC0=(286.405-S0RT(53671.0-3.1027*(C-152.45)**2))*.OOOOODl RETJRN END

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Subroutine DIEQS

First subroutine PREDI is called to determine the value of the 6 index. Then, the length of the filter run (TR) and the thickness of the precoat and filter cake (THICK) at the end of the filter run are calculated. The length of the filter run is calculated as the length of the filtering cycle (TF) plus the time required to backwash and precoat the filter (BWT).

SUBROUTINE DIEQS COMMON IN(150), ANS(13,10), B(4), AMORT, AREA, BETA, BWT, CD, ICDT,CDS,CDF,CnE,CP3,CF,CL,CM,CB,C3PFR,CT3TL,CS,EFF, 2 FACT 5,5,GP,%d,HP,HC,KPIT,KPOT,PHI,Qi,QS,QF,QGPM,QqCl, 3QMGDP,RF,R3,RS,\$TGqA,TH,THI,THS,THF,TR,THICK,UQ,VIS, 4W,XI,XLP

```
CALL PREDI(FACTR, BETA)
  PHI=2.0*JQ*GW*CD*.000001/5P 
  SIGMA=U3*U0*VIS*BETA*CD/G 
  XLP=W/GP 
  HP=JO*VI$*XI*W/G 
  HC=TH-HP 
  IF(RS)2,1,2 
1 TF=HC/SIGMA 
  TP=TF+BWT 
  THI:K=XLP+PHI*TF/2.0 
  GO TO 3 
2 RO=RS+XLP 
  TF=R0*R0*(EXP(HC*PHI/(RS*SIGMA))-1.0)/(RS*PHI)
  TR=TF+3WT 
  THT:<=SQRT(%3*R0+RS*PHI*TF)-RS 
3 RETJRN 
  END
```
Subroutine PREDI

This subroutine calculates the value of β using the prediction equation. ξ is included as an additional variable however it is recommended that it be excluded and b^{α} read in as 0. The argument DUMMY is the 6 multiplication factor (FACTR).

 $\sim 10^{-1}$

```
SUBROUTINE PREDI(DUMMY,PRED) 
  COMMON IM(150),ANS(13,10),B(4),AMORT,AREA,BETA,SWT,CO, 
 ICDI,CDS,CDF,CDE,CPT,CF,CL,CM,CB,COPER,CT]TL,CS,EF=, 
2FACTR,G,GP,GW,HP,HC,KPIT,<P0T,PHI,QI,QS,QF,QGPM,QM30, 
3QMG3P,RF,RO,RS,SIGMA,TH,THT,THS,THF,TR,THICK,UQ,VIS, 
4W,XI,XLP 
  PRED=nUMMY*13.0**B(1) 
  IF(B(2))1,2,11 PRED=PRED*CS**B(2) 
2 IF(B(3))3,4,3 
3 PRED=PRFD*CD**B(3) 
4 IF(B(4))5,6,55 PRED=PRED*XI**B(4)
6 RETJ9X 
  END
```
Subroutine COSTS

Subroutine CBAKW is called to calculate the required filter area. Each of the cost subroutines are then called to calculate the various costs in \$/month. The operating and total cost is then calculated. Maintenance cost (CM) and backwashing cost (CB) are both equal to 0 since maintenance cost is included with labor cost (CL) and the costs ascribed to backwashing are accounted for by the increase in filter area required to produce water for backwashing (see subroutine CBAKW) .

SUBROUTINE COSTS COMMON.IN(150),ANS(13,10),B(4),AMORT,AREA,BETA,BWT,CD, 1C5Î,ens,CDF,:OE,CPO,CF,CL,Gq,CB,COPER,CT3TL,CS,EPF, 2 FACTR, S, $3P$, $3W$, HP, HC, KPIT, ζ P 0 T, P HI, Q I, QS , QF , QGP M, $2W3D$, 30MGDP,RF,R0,RS,SIGMA,TH,THI,THS,THF,TR,THICK,UQ,VIS, 4W,XI,XLP CALL CBAKW(2) CALL CFUST(2) CALL CLABR{2) CALL CDIAT(2) CALL CMAIN12) CALL CPDWP(?) COPFR=CDE+CM+CPO+CL+CB CTOTL=CF+COPER RETURN END

Subroutine CBAKW

When subroutine CBAKW is called from subroutine READR, the argument L is equal to 1 and subroutine VALUE is called to determine the gallons of backwash water required per sq ft (BWGSF) and the time in minutes required to backwash and precoat the filter (TBW) punched on the data card (index number 20). When subroutine CBAKW is called from subroutine COSTS, the argument L is equal to 2 and the required filter area (AREA) and flow rate in MGD (QMGDP) required to meet both demand and backwashing requirements are calculated using the iterative procedure presented on page 169. RPD is the number of filter runs that can be made per day and TMFD is the time in minutes per day that the filter is actually filtering.

SUBROUTIMF CBAKW(L) COMMON IN(150), ANS(13,10), B(4), AM3RT, AREA, BETA, BWT, CD, icni,CDS,CDF,CDS,CPO,CF,CL,:q,CB,COPER,CT0TL,CS,EFC, 2FACTR,G,SP,GW,HP,HC,KPIT,KPOT,PHI,QI,0S,QF,QGPM,Q^GD, %F,R3,RS,SIGMA,TH,THI,THS,THF,TR,THICK,UO,VIS, 4W,XI,XLD $IF(L-1)1,1,2$ 1 CALL VALUE(26,BWGSF) F1=BWGSF CALL VALUF(0,TBW) BWT=TBW/60. $E2 = TRW$ RETURN 2 RPD=24./TR TMFD=1440.-RPD*F2 QMGDP=0M3D 3 OGDM=9MGDP*1000000./TMFO AREA=0GPy/(U0/8.02)

3MGDP=QMSD+RPD*AREA*F1/100D030. QGPM=QMGDP*1000000./TMFD CAREA=QGPM/((JQ/8.02) 3MG3P=QMSO+RPD*CAREA*F1/1030330. F3=0.01*ARFA F4=CAREA-AREA IF(F3-F4)3,3,4 4 AREA=CAREA $CB=0.0$ RETURN END

Subroutine CFUST

When subroutine CFUST is called from subroutine READR, the argument L is equal to 1. In this case, the input cards containing the first cost per unit area versus filter area data are read in and interpreted by calling subroutine VALUE. Logarithms of the filter area are stored in array A and the corresponding logarithms of the unit first cost are stored in array Z. LIMIT is the element of arrays A and Z which contain the last log area and log unit first cost values.

When subroutine CFUST is called from subroutine COSTS, the argument L is equal to 2. In this case, subroutine YINT is used to determine the first cost per unit area which corresponds to the required filter area (AREA) and the amortized first cost is calculated in \$/month.

```
SUBRGUTTNE CFUST(L) 
    \texttt{COMMON} IN(150), ANS(13, 10), B(4), AMORT, AREA, BETA, BWT, CD,
   lCDI,:DS,COF,COE,CP],CF,CL,COPER,CT3TL,CS,EF=, 
   ?FACTR,G,GP,Sd,HP,HC,KPIT,<P0T,PHI,QI,0S,gF,gGPM,0q3D, 
   30MG3P,%F,R3,R$,SIGMA,T4,THT,THS,THF,TR,T4ICK,U0,VIS, 
   4W,XI,XLP 
    DIMENSION A(50),Z(50)
    IF(L-1)1,1,41 DO 2 1=1,50 
    READ(KPIT, 100) (IN(J), J=1, 80)
100 F0RMAT(80A1) 
    WRITE(KPOT, 200) (IN(J), J=1, 80)
?00 F0R^AT(1X,R0A1) 
    CALL VALUE(26,TEMP) 
    A(I) = ALOG(TEMP)CALL VALUE(0,TEMP) 
    Z(I) = ALJG(TEYP)IF(IN(6J-IN(95))3,2,3 
  2 CONTINUE 
  3 LIMIT=I 
    RETURN
  4 TEMP=ALOG(AREA) 
    RF=1.+(UQ-8.1/40.TEMP=YINT(LIMIT,TEMP,A,Z) 
    CF=EXP(TEMP)*AREA*AMORT*RF 
    RETURN 
    END
```
Subroutine CLABR

Subroutine CLABR is similar to subroutine CFUST. ' When it is called from subroutine READR, L equals 1 and the labor cost versus filter area data are read in and interpreted. When it is called from subroutine COSTS, L equals 2 and the cost of labor per unit filter area per month which corresponds to the required filter area is determined, and the cost of labor in \$/month is calculated.

```
SUBROUTINE CLABR(L) 
    COMMON IN(150),&NS(13,10),B(4),AMORT,A%EA,BFTA,BWT,CD, 
   1 GDI,CDS,CDF,CDE,CPO,CF,CL,CM,CB,COPER,CTOTL,CS,EFP, 
   2FACTR,S,GP,SW,HP,HC,KPIT,KPOT,PHI,QI,QS,QE,QGPM,QMGD,
   33MG0P,RF,R0,RS,SIGMA,TH,THI,THS,THF,TR,THICK,UQ,VIS, 
   4W,XI,XLP 
    DIMENSION A(50),Z(50) 
    IF(L-1)1,1,41 on 2 1=1,50 
    READ(KPIT,100)(IN(J),J=1,80) 
100 FORMAT(ROAl) 
    WRITE(KPOT, 200) (IN(J), J=1, 80)
200 FORMAT(1X,80A1) 
    CALL VALUE(26,TEMP) 
    A(I)=ALN3(TEMP) 
    CALL VALUE(0,TEMP) 
    Z(I) = ALOG(TEMP)IF(IN{6)-IN(95))3,2,3 
  2 CONTINUE 
  3 LIMIT=I 
    RETURN 
  4 TEMP=ALOG(AREA) 
    TEMP=YINT(LIMIT,TEMP,A,Z) 
    CL=EXP(TEMP)*AREA*RF 
    RETURN 
    END
```
Function YINT

This subroutine is used to determine the value (YINT) of the first cost in \$/sq ft, or the cost of labor in \$/sq ft per month, which corresponds to the required filter area. This was done by linear interpolation between points read in from the first cost, or labor cost, versus area curve. X is the logarithm of the required filter area, array AX contains the logarithms of the values of filter area, and array AY contain the logarithms of the corresponding cost per unit area values. Logarithms are used to make the curve more linear.

If the required filter area is less than the lowest area for which a cost value was read in, YINT is set equal to the cost value for the lowest area read in and a caution statement is printed. If the required filter area is greater than the largest area for which a cost value is read in, YINT is set equal to the cost value for the largest area read in and a caution statement is printed. The original version of POPO (27) contained an error such that if the required filter area was exactly equal to any of the areas for which a cost value was input, YINT was set equal to the cost value for the largest area read in. The present program has been changed to correct this mistake.

FUNCTION YIMT(LIMIT,X,AX,AV) DIMENSION AX(50),AY(50) $IF(X-AX(1))6, 1, 2$ $1 \quad YINT=AY(1)$ RETURN 2 on 3 1=2,LIMIT $IF(X-AX(I))$ ^{4,5,3} 3 CONTINUE YINT=AY(LIYIT) WRITF(KP0T,100) 100 FORMAT(10X, *** CAUTION ** AREA ABOVE RANGE OF COST DATA*) RETURN 4 $J=I-1$ YINT=AY(J)+(X-AX(J))*(AY(I)-AY{J))/(AX(1)-AX(J)) RETURN 5 YINT=AY(I) RETURN 5 YINT=AY(1) WRIT^c(KPOT,200) 200 FORMAT(10X, *** CAUTION ** AREA BELOW RANGE OF COST DATA') RETURN END

Subroutine CDIAT

When subroutine CDIAT is called from subroutine READR, the argument L is equal to 1 and subroutine VALUE is called to determine the price of filter aid (UCDE) punched on the input card (index number 16). When subroutine CDIAT is called from subroutine COSTS, the argument L equals 2 and the cost of filter aid (CDE) in \$/month is calculated.

SUBRIUTINE CDIAT(L) COMMON IN(150), ANS(13,10), B(4), AMORT, AREA, BETA, BWT, CD, irOT,205,CDF,CDE,CPO,CF,CL,Cy,CB,COPER,CT3TL,CS,EF=, 2FACTq,G,GP,Gd,HP,HC,KPTT,<P0T,PH7,QI,QS,3F,QGPM,9M3D, 3QMG3P,RF,RD,RS,SIGMA,H,THI,THS,THF,TR,THIC<, JQ,\/IS, 4W,XI,XLP **1 CALL VALUF(26,UCDE)** $F1= 24. *30.4 / 2000.$ F2=8.33*30.4/2000. RETURN 2 PREDE=F1*W*ARFA/TR BFDE=F2*CD*Q^GDP CDE=JCOE*(PREDE+BFDE) RETURN

END

Subroutine CMAIN

In the present form of POPO, maintenance costs are included with the cost of labor. Therefore, the cost of maintenance (CM) is set equal to 0. The subroutine is included in case it becomes desirable to calculate maintenance cost separately from labor cost.

```
SUBROUTINE CMAIN(L) 
  :GMMON IN(150),&NS(13,101,B(4),AMORT,AREA,BETA,BWT,CO, 
 1 C DÎ, C DS, CDF, CDE,Ct>Q,CF,CL, CM, CB, COPER, CTOTL,CS,EFF, 
 2FACT3,G,GP,3W,HP,HC,KPIT,KP0T,PHI,QI,QS,QF,QGPM,QMGO, 
 30MGDP,RF,RO,RS,SIGMA,TH,THI,THS,THF,TR,THICK,UQ,VIS, 
 4W,XI,XLP 
  IF(L-1)1,1,2 
1 RETURN 
2 CM=0.0 
  RETURN 
  END
```
Subroutine CPOWR

When subroutine CPOWR is called from subroutine READR, the argument L is equal to 1 and subroutine VALUE is called to determine the cost of power in ζ/κ wh (PP) punched on the data card (index number 18). When subroutine CPOWR is called from subroutine COSTS, the argument L is equal to 2 and the power cost in \$/month (CPO) is calculated.

Sales Corp. SUBROUTINE CPOWR(L) **COMMON IN(150),ANS(13,10),B(4),AMORT,AREA,BETA,SWT,CD, ICOI, CDS,CDF,CDE,CPO,CF,CL,CM,CB,COPER,CTOTL,CS,EF = , 2FACT5,G,GP,3W,HP,HC,KPIT,KP0T,PHI,QI,QS,QF,QGPM,gMGD, 30MGDP,RF,RO,SS,SIGMA,TH,THI,TH\$,THF,TR,THICK,UQ,VIS, 4W,XI,XLP IF(L-1)1,1,2 Î CALL VALUE!26,P%) CONST=(PP/100.)*8.33*30.4/2.655 RETURN ? CPn=:ONST*TH*QMGDP/EFF RETURN END**

Subroutine STRES

Subroutine STRES is called from the MAIN program after the total cost for a particular combination of filtration rate, terminal head loss, and body feed rate has been determined. This total cost is then compared to the 10 total cost values stored in the 8th *row* of the ANS array. If it is less than any of these, it is stored in the proper place in ANS such that the 10 total cost values in the array are arranged in ascending order.

SUBROUTINE STRES COMMON IN(150), ANS(13,10), B(4), AMORT, AREA, BETA, BWT, CD, 1 CDI,CDS,CDF,CDE,CP3,CF,CL,CM,CB,C0PER,CTDTL,CS,EFF, 2FACTR,G,SP,3W,H0,HC,KPIT,<POT,PHI,QI,3S,QF, QGP^, 0430, 3QMGDP,RF,RO,RS,SIG%A,TH,THI,THS,THF,TR,THICK,UQ,VIS, 4W,XT,XLP $LIMI$ ^{T=10} DO 1 K=L,LIMIT $IF(CTOTL-ANS(8,K))2,1,1$ 1 CONTINUE RETURN ? J=LIMTT 3 IF(J-K)6,6,4 4 L=J-1 DO 5 T=1,13 5 ANS(I,J)=ANS(I,L) $J = L$ GO T7 3 6 ANS($1, K$) = UQ $ANS(2, K) = TH$ $ANS(3, K) = CD$ ANS(4,K)=BETA $ANS(5, K) = TR$ $ANS(6, K) = AREA$ ANS(7,K;=THICK ANS(8,K)=CT0TL $ANS(9, K) = CF$ ANS(10,K)=C0PER

 $ANS(11,K)=CL+CM$ $ANS(12, K) = CP0$ $ANS(13,K)=CDE$ RETURN END

Subroutine OUTPT

This subroutine is called from the MAIN program to print the final results for each value of β . Note that the cost values presented in \$/MG are based on the demand flow rate (QMGD) and not the flow rate required to meet both demand and backwashing requirements. After the results are printed out, the 8th row of the ANS array is reinitialized with large numbers.

```
SUB9 3UTTNE TUTPT 
    COMMON TN(150),ANS(13,10),B(4),AMORT,AREA,BETA,BWT,CD,
   ICDT,COS,CDF,CDE,CPO,CF,CL,:%,CB,COPER,CriTLfCS,EPF, 
   2FACTR,G,SP,GW,HP,HC,KPIT,<PGT,PHÎ,QT,3S,3F,QGPM, a^GD, 
   3QMGDD,RF,R0,R$,$IGMA,TH,THI,THS,THF,TP,THICK,U3,VIS, 
   4W,XI,XLP 
    T=FACTR*100.0 
    WRTTF(KP0T,100)I 
100 FOR%AT('0',28X,'8ETA INDICES =',I4,' PERCENT OF PREDICTED VALUES*) 
    DO 2 1=1,10 
    ANS(l,I)=ANS(l,T)/8.02 
    J=ANS(2,1)K=ANS(3,1)L = AVS(4, I)/10000.0w=ANS(6,1) 
    ANS(7, I) = ANS(7, I)*12.NN=AVS(S, I)on 1 KK=8,13 
  1 ANS(KK,I)=AN$(KK,I)/(0MGD*30.4) 
  2 HPIT = (KPOT, 200) ANS(1, I), J, K, L, ANS(5, I), M, (ANS(N, I), N=7, 13), NM200 F1R%4T(F5.2,I6,I5,I8,F7.1,I6,F7.2,' *',6^6.1,' *',I8) 
    BIG=1OC^OOO.**4 
    DO ? L=l,10 
  3 ANS(8,L)=BIG 
    RETJRN 
    END
```
Example input and output

The input data and final results for two POPO jobs are presented in this section. The data is presented here for demonstration purposes only.

Job 1 illustrates the use of POPO for optimizing the design of a proposed plant. The design suspended solids concentration is 7.5 mg/1 of iron and the minimum amount of body feed found to still produce an incompressible cake was about 20 mg/1. Therefore, the initial value of the body feed rate was set at 20 mg/1. The final value of the terminal head loss is limited to 150 feet. Values of the initial and final filtration rate, final body feed rate, and initial terminal head loss were chosen so that the optimum design conditions would be within the range of the conditions for which cost calculations were made.

Job 2 illustrates the use of POPO for optimizing the operation of an existing plant. In this example, both the filtration rate and the terminal head loss are fixed. Therefore, only the body feed rate was varied in the optimization calculations.

Job 1 and Job 2 were both processed during the same computer run. Job 2 illustrates that it is necessary to input only the data that is different from that of the previous job.

POPG — PROGRAM FOR OPTIMIZATION OF PLANT OPERATION

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 $\label{eq:2} \frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^$

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POPO — PROGRAM FOR OPTIMIZATION OF PLANT OPERATION

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0 JOB 2. LIME-SODA ASH PROCESS EFFLUENT
1 DESIGN FLOW 4.5 MGD 1 DESIGN FLOW 4.5 MGD
5 PLANT LIFE 30 YEARS 5 PLANT LIFE 30 YEAR
5 SOLIDS (CS) 30 SS 9PM 6 SOLIDS (CS) 8.5 PPM
8 TEMPERATURE 65 DEGREES F 8 TEMPERATURE 65 DEGREES F 9 PRECOAT WEIGHT 11 SEPTUM DIAMETER
12 BETA PREDICTION $10.20/1.43/-3.29/0$
0.73 13 UNIT FLOW RATE 0.73 GSFM
14 BODY FEED 10/2/30 PPM 14 BODY FEED 10/2/30 PPM
15 TERMINAL HEAD 25 FT 15 TERMINAL HEAD 25
16 DIATOMITE COST 69 \$/TON 16 DIATOMITE COST
20 BACKWASH COST 6, 30 GAL/SF, MIN BEGIN

Manual calculations

Data

Example Number 1 - Iron removal

The filtration and cost data for this example are printed as part of the computer output (p. 416). The data from which the B prediction equation was developed was collected by Hall and Hawley (see Appendix A, Table 24, Runs 32-37) who filtered University tap water to which ferrous sulfate was added. Celite 503 filter aid was used for both precoat and body feed.

Cost calculations are made below for the case where;

 $q = 1.00$ gpm/sq ft H_+ = 150 ft C_F = 40 mg/1

Calculations

1. S index

The β prediction equation is:

$$
\beta = 10^{9.33} (C_S/C_F)^{1.95}
$$

Therefore, since $C_S = 7.5$ mg/1 and $C_F = 40$ mg/1, then:

$$
\beta = 10^{9.33} (7.5/40)^{1.95}
$$

$$
= 10^{9.33} (0.1875)^{1.95}
$$

and taking logarithms of both sides of the equation:

 $log \ \beta = 9.33 + 1.95(-0.7270)$

$$
= 9.33 - 1.4177
$$

$$
= 7.9123
$$

and :

$$
\frac{\beta = 8.171 \times 10^{7} \text{ ft}^{-2}}{2. \text{ Filter run length}}
$$

H_p = qvξw/g (5)

where :

$$
q = 1.00 \text{ gpm/sq ft}
$$

$$
= 1.00 (8.02) ft/hr
$$

$$
= 8.02 ft/hr
$$

$$
v = \frac{\mu}{\gamma_w}
$$

where :

$$
\mu = 1.2028 \times 10^{-2} \text{ poise} \qquad \text{from handbook (21)}
$$

= (1.2028 x 10⁻²)242 lb/hr ft
= 2.911 lb/hr ft
 γ_w = 62.39 lb/cu ft from handbook (21)

 $\mathcal{L}^{\text{max}}_{\text{max}}$

therefore :

$$
v = \frac{2.911}{62.39} = 4.666 \times 10^{-2}
$$
 sq ft/hr
\n $\xi = 1.95 \times 10^9$ ft/lb
\n $w = 0.15$ lb/sq ft

and:

$$
g = 32.2 \text{ ft/sec}^2
$$

$$
= 417.3 \times 10^6 \text{ ft/hr}^2
$$

Therefore :

$$
H_{p} = \frac{(8.02) (4.666 \times 10^{-2}) (1.95 \times 10^{9}) (0.15)}{417.3 \times 10^{6}}
$$

\n= 0.26 ft
\n
$$
H_{c} = H_{t} - H_{p}
$$

\n= 150 - 0.26
\n= 149.74 ft
\n
$$
t_{f} = \frac{(e^{H_{c} \phi/R_{s} \sigma} - 1)R_{o}^{2}}{R_{s} \phi}
$$

\nwhere:
\n
$$
\phi = 2q\gamma_{w}C_{F}(10^{-6})/\gamma_{p}
$$

\n= $\frac{2 (8.02) (62.39) (40) 10^{-6}}{15}$
\n= 2.669 x 10⁻³ ft/hr
\n
$$
= \frac{(1 \text{ inch}) (1 ft/12 \text{ inches})}{(15)(1600)}
$$

$$
= 4.167 \times 10^{-2} \text{ ft}
$$

\n
$$
\sigma = q^2 v \beta C_F / g
$$

\n
$$
= \frac{(8.02)^2 (4.666 \times 10^{-2}) (8.171 \times 10^7) 40}{417.3 \times 10^6}
$$

2

= 23.506 **ft/hr**

 $R_0 = R_s + L_p$

where :

 $\frac{1}{\sqrt{2}}$, $\frac{1}{2}$

$$
L_p = w / Y_p
$$

 $\omega_{\rm 25mm}$

 $\bar{\mathbf{A}}$

 \sim
$$
= \frac{0.15 \text{ lb/sq ft}}{15 \text{ lb/cu ft}}
$$

$$
= 0.01 \text{ ft}
$$

therefore :

$$
R_o = 0.04167 + 0.01
$$

= 5.167 x 10⁻² ft

Now:

$$
\frac{H_C \phi}{R_S \sigma} = \frac{(149.74) (2.669 \times 10^{-3})}{(4.167 \times 10^{-2}) (23.506)}
$$

= 0.4080

and, therefore:

$$
t_f = \frac{(e^{0.4080} - 1) (5.167 \times 10^{-2})^2}{(4.167 \times 10^{-2}) (2.669 \times 10^{-3})}
$$

 $= 12.1$ hr

The time required to backwash and precoat the filter is estimated to be 30 min or 0.5 hr. Therefore the total length of the filter run is 12.6 hr.

 ϵ

3. Filter area

QGPM' =
$$
\frac{QMGD' \times 10^6}{1440 - n \text{ (BWT)}}
$$

where :

$$
n = \frac{24}{12 \cdot 6} = 1.91 \text{ filter runs per day}
$$

and:

BWT = 30 min per filter run

so that:

 ~ 100

 $1440 - n$ (BWT) = 1383 min

If it is assumed that $QMGD' = QMGD$, then:

$$
\text{QGPM'} = \frac{1 \times 10^6}{1383}
$$

$$
= 723.2 \text{ gpm}
$$

and:

Area =
$$
\frac{QGPM'}{q}
$$

=
$$
\frac{723.2 \text{ gpm}}{1.00 \text{ gpm/sq ft}}
$$

= 723.2 sq ft

$$
QMGD' = QMGD + \frac{n \text{ (Area) BWGSF}}{10^6}
$$

where :

 $BWGSF = 10 gal/sq ft$

therefore :

$$
QMGD' = 1 + \frac{1.91 (723.2) 10}{10^6}
$$

= 1.0138 MGD

If the preceding steps are repeated, a corrected value of Area is obtained:

 $\mathcal{L}(\mathcal{L}_\mathbf{p})$ and $\mathcal{L}_\mathbf{p}$

$$
QGPM' = \frac{1.0138 \times 10^6}{1383}
$$

= 733.2 gpm
Area = $\frac{733.2 \text{ gpm}}{1.00 \text{ gpm/sq ft}}$
= 733.2 sq ft

This value of Area is more than one percent greater than the preceding value of Area, Therefore, another iteration is required.

$$
QMGD' = 1 + \frac{1.91 (733.2) 10}{10^6}
$$

= 1.0140 MGD

$$
QGPM' = \frac{1.0140 \times 10^6}{1383}
$$

= 733.4 gpm
Area =
$$
\frac{733.4 gpm}{1.00 gpm/sq \text{ ft}}
$$

= 733.4 sq ft

This value is within one percent of the preceding value of Area. Therefore, the required filter area is 733 sq ft and QMGD' is 1.0140 MGD.

4. First cost

From a plot of first cost per unit of filter area versus filter area, the first cost per unit of filter area is found

425

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to be \$107/sq ft. Therefore, the total first cost is:

$$
TFC = (107 \, \text{S/sq ft}) \quad (733 \, \text{sq ft}) \\
= \, \$78,458
$$

The first cost data were gathered for a filtration rate of 1 gpm/sq ft. Therefore, in this example, the filtration rate factor is equal to 1.0 since the filtration rate is 1.00 gpm/sq ft.

The first cost is amortized over the design life of the plant by the equation:

CF per year = TFC
$$
\{\frac{i[(1-i)^n - SV/100]}{(1-i)^n - 1}\}
$$
 (44)

which is equivalent to:

$$
CF per year = TFC \frac{(caf' - SV/100)}{caf}
$$

where :

 $\text{caf'} = \text{single payment compound amount factor} (34)$

caf = uniform series compound amount factor (34) From a handbook of interest tables for $i = 4$ % and $n = 25$ years :

 $caf' = 2.6658$

and :

 $cat = 41.6459.$

Therefore :

CF per year =
$$
\frac{$78,468}{41.6459} \cdot \frac{(2.6658 - 15/100)}{41.6459}
$$

$$
= $4740 per year
$$

and:

CF per month =
$$
\frac{\$4740}{12} = \frac{\$395 per month}{}
$$

5. Labor and maintenance cost

From a plot of labor and maintenance cost in \$/sq ft per month versus filter area, the labor and maintenance cost was found to be 0.52 \$/sq ft per month. Thus;

CL + CM = (0.52) (733) RF
= (381
$$
\frac{\text{S}}{\text{month}}
$$
)1.0
= \$381 per month

6. Filter aid cost

$$
PFA = w(Area)N \qquad (45)
$$

where:

$$
N = \frac{(24 \times 30.4) hr/month}{12.6 hr/run}
$$

= 58 runs/month

therefore :

PFA = (0.15) (733) (58) = 6380 lb/month BFA = Cp(QMGM0')8.33 (46) = 40(30.**4 X** 1.0140)8.33 = 10,271 lb/month

The total cost of filter aid per month is:

$$
CFA = \frac{(6380 + 10271)}{2000} (5100/ton)
$$

\n
$$
= \frac{5833 \text{ per month}}{2000}
$$

\n7. Power cost
\n
$$
P = \frac{QMGMO' \times H_t}{E} \frac{8.33}{2.655}
$$

\n
$$
= \frac{(30.4 \times 1.0140)(150)}{0.70} \frac{8.33}{2.655}
$$

\n
$$
= 20,725 \text{ kWh/month}
$$

\nTherefore:
\n
$$
CP = 20725 \times 0.02 \text{ S/kwh}
$$

\n
$$
= \frac{6415 \text{ per month}}{8. \text{ Total and operating costs}}
$$

\n
$$
CDEF = CL + CM + CFA + CP
$$

\n
$$
= 381 + 833 + 415
$$

\n(49)

 $= 1629 per month

and:

$$
\text{CTOTL} = \text{CF} + \text{COPER} \tag{50}
$$
\n
$$
= 395 + 1629
$$
\n
$$
= \$2024 \text{ per month}
$$

To convert these costs to \$/MG of finished water, **they** should be divided by 30.4 MG/month.

A comparison of results of manual and computer calculations is made below.

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2.$

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APPENDIX G

Abbreviations

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Abbreviation Meaning

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Notation

431

432

 $\frac{1}{2}$.

